Active flywheel control for hybrid vehicle

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Abstract – **In the paper, the authors propose a novel control strategy of torque ripple on hybrid vehicle. The combustion engine ripple's are reduced by using an active filter and an AC machine which is mounted on the crankshaft to generate on inverse torque sequence. The control strategy is based on a multi-objectives state feedback synthesis. A complete modelling of the hybrid propulsion of the vehicle is achieved. Simulation results highlight the interest of the control scheme.**

1. Introduction

In recent years, noise and vibration of automotive engines is becoming an integral part of the design process. Torque ripple, especially at low speeds is still one of the important source of vibration. This torque ripple comes from internal combustion engine nature and produces undesirable acoustic noise. Until now, only passive solutions are used to reduce these ripple.

Active filtering of torque ripple reduces torque pulsation and thus provides an original way of overcoming the related problem. This technique requires an actuator coupled with the internal combustion engine. In the case of a hybrid vehicle, one has an actuator allowing the active compensation of the torque ripple. A permanent magnetic synchronous machine replace the flywheel that is normally used to smooth the oscillation torque of the internal combustion engine.

This paper is organized as follows. Section 2 gives the mathematical modeling of the system. Section 3 deals with the state feedback design controller. The controller is obtained using Linear Matrix Inequalities framework. Simulation results are presented in section 4.

2. Mathematical modeling

2.1 Principle

The system is depicted on figure 1.

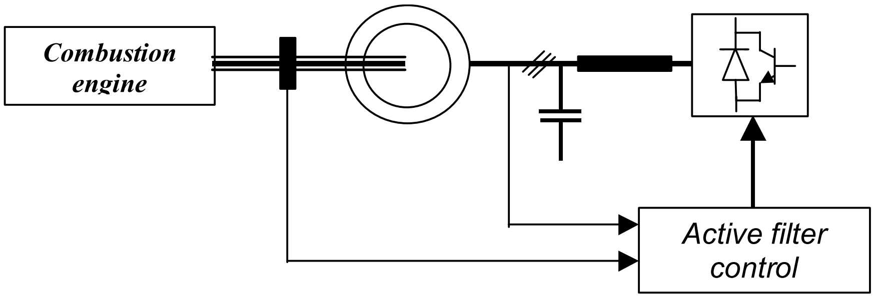

Fig. 1. Principle

The permanent magnetic synchronous machine is coupled to the internal combustion engine from the crankshaft.

Active filter is made of parts of an inverter, a *LC* passive filter and a permanent synchronous machine (fig. 2).

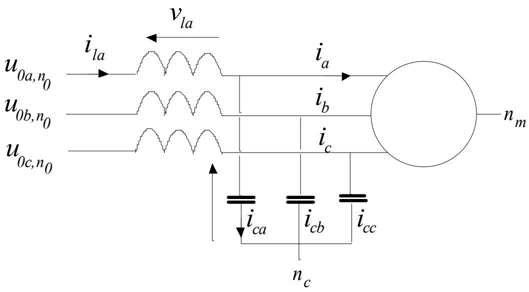

Fig. 2. Active filter

We assume that $u_{n_0} = u_{n_c} = u_{n_m}$.

Voltage outputs of the inverter present high frequencies components, inherent in the technology of the power electronics. These high frequencies components are found partly on the torque of the electric actuator. To eliminate them, a *LC* passive electrical filter is inserted between the inverter and the electrical motor. To avoid an useless dissipation of energy, the filter has no damping resistor, and tends to destabilize the system. One of the roles of the control laws will be to inhibit the damping between the passive filter capacitor and the equivalent reactance of the machine.

2.2 System modeling in *dq* frame

Denoting by P_θ and P_θ^{-1} the direct and inverse rotation matrices, and assuming that $x_a + x_b + x_c = 0$, the direct and inverse Park transforms are defined as:

$$\begin{bmatrix} x_d \\ x_q \end{bmatrix} = P_\theta \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \quad , \quad \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = P_\theta^{-1} \begin{bmatrix} x_d \\ x_q \end{bmatrix} \tag{1}$$

where

$$P_\theta=\sqrt{\frac{2}{3}}\begin{bmatrix} cos(\theta) & cos(\theta-2\frac{\pi}{3}) & cos(\theta+2\frac{\pi}{3}) \\ -sin(\theta) & -sin(\theta-2\frac{\pi}{3}) & -sin(\theta+2\frac{\pi}{3}) \end{bmatrix} \text{ and } P_\theta^{-1}=\sqrt{\frac{2}{3}}\begin{bmatrix} cos(\theta) & -sin(\theta) \\ cos(\theta-2\frac{\pi}{3}) & -sin(\theta-2\frac{\pi}{3}) \\ cos(\theta+2\frac{\pi}{3}) & -sin(\theta+2\frac{\pi}{3}) \end{bmatrix}.$$

From (1), and denoting by $\omega=\dot{\theta}$, voltages and currents in capacitors are given by

$$P_\theta^{-1}\begin{bmatrix} i_{dc} \\ i_{qc} \end{bmatrix} = C_a \frac{d}{dt}\left(P_\theta^{-1}\begin{bmatrix} v_d \\ v_q \end{bmatrix}\right) = C_a\left(\frac{d}{dt}\left(P_\theta^{-1}\right)\begin{bmatrix} v_d \\ v_q \end{bmatrix} + P_\theta^{-1}\frac{d}{dt}\begin{bmatrix} v_d \\ v_q \end{bmatrix}\right)$$

then

$$\begin{bmatrix} i_{dc} \\ i_{qc} \end{bmatrix} = C_a\left(P_\theta \frac{d}{dt}\left(P_\theta^{-1}\right)\begin{bmatrix} v_d \\ v_q \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} v_d \\ v_q \end{bmatrix}\right)$$

$$= C_a\left(\begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix}\begin{bmatrix} v_d \\ v_q \end{bmatrix} + \begin{bmatrix} \dot{v}_d \\ \dot{v}_q \end{bmatrix}\right)$$

In the same way, voltages and currents in the self are given by

$$\begin{bmatrix} i_{dl} \\ i_{ql} \end{bmatrix} = L\left(\begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix}\begin{bmatrix} i_{dl} \\ i_{ql} \end{bmatrix} + \begin{bmatrix} \dot{i}_{dl} \\ \dot{i}_{ql} \end{bmatrix}\right)$$

In the *dq* frame, the permanent magnetic synchronous machine equations are given by

$$\begin{aligned} \dot{i}_d &= -\frac{R}{L_d} i_d + \frac{L_q}{L_d} p\omega i_q + \frac{1}{L_d} v_d \\ \dot{i}_q &= -\frac{R}{L_q} i_q - \frac{L_d}{L_q} p\omega i_d + \frac{1}{L_q} v_q - \frac{\lambda}{L_q} p\omega \end{aligned} \tag{2}$$

The electrical equations of the active filter are as follows:

$$\begin{aligned} u_{d0} &= L(\dot{i}_{dl} - \omega i_{ql}) + v_d \\ u_{q0} &= L(\dot{i}_{ql} + \omega i_{dl}) + v_q \\ i_{dl} &= i_{dc} + i_d \\ i_{ql} &= i_{qc} + i_q \end{aligned} \tag{3}$$

From (2) and (3), the active filter dynamics are described by:

$$
\begin{aligned}
\dot{i}_d &= -\frac{R}{L_d} i_d + \frac{L_q}{L_d} p\omega i_q + \frac{1}{L_d} v_d \\
\dot{i}_q &= -\frac{R}{L_q} i_q - \frac{L_d}{L_q} p\omega i_d + \frac{1}{L_q} v_q - \frac{\lambda}{L_q} p\omega \\
\dot{v}_d &= -\frac{1}{C_a} i_d + \omega v_q + \frac{1}{C_a} i_{dl} \\
\dot{v}_q &= -\frac{1}{C_a} i_q - \omega v_d + \frac{1}{C_a} i_{ql} \\
\dot{i}_{dl} &= -\frac{1}{L} v_d + \omega i_{ql} + \frac{1}{L} u_{d0} \\
\dot{i}_{ql} &= -\frac{1}{L} v_q - \omega i_{dl} + \frac{1}{L} u_{q0}
\end{aligned} \tag{4}
$$

2.3 Combustion engine modeling

The mechanical model of a four cylinder combustion engine is given by:

$$
\begin{aligned}
\dot{\theta}_0 &= \omega \\
J\dot{\omega} &= \sum_{k=0}^{3} \left(C_{pk} + C_{ik}\right) - T_{em} - T_1
\end{aligned} \tag{5}
$$

where C_{pk} denotes the torque generated by the pressure in the kth cylinder, C_{ik} is the torque generated by oscillating masses and connecting rods, T_{em} is the opposite torque of the synchronous machine, and T_1 is the exogenous load torque. Mathematical models are given by :

$$
\begin{aligned}
C_{pk} &= P_r(\theta_k)\left(r\,cos(\theta_k) + l\sqrt{1-\lambda_m^2\, sin^2(\theta_k)}\right)tan(\varphi_k) \\
C_{ik} &= \left(m_a + m_p\right)\omega^2 r\left(cos(\theta_k) + \lambda_m\, cos(2\theta_k)\right)\left(r\,cos(\theta_k) + l\sqrt{1-\lambda_m^2\, sin^2(\theta_k)}\right)tan(\varphi_k) \\
T_{em} &= \frac{3}{2} p\left(\left(L_d - L_q\right) i_d i_q + \lambda i_q\right)
\end{aligned} \tag{6}
$$

where $\theta_k = \theta_0 + k\pi$ and $sin(\varphi_k) = -\lambda_m\, sin(\theta_k)$. $P_r(\theta_k)$ is the upward thrust on the kth stroke. $\lambda_m = \frac{r}{l}$ where $l = 0.1m$ is the length of the connecting rods and $r = 0.29m$ the course of the stroke. m_a=$0.235kg$ represents the mass of the connecting rods and $m_p = 0.64kg$ the mass of the stroke.

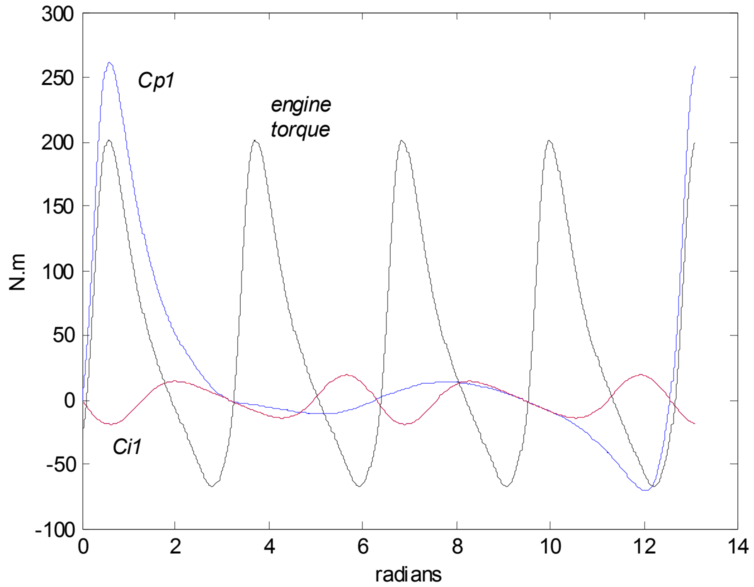

Figure 3: inertial torque C_{i1} (dotted lines), torque C_{p1} and total torque of the combustion engine

3. Controller design

As previously mentioned, the control purpose is to minimize the oscillation torque of the combustion engine, that is composed of two components C_{pk} and C_{ik}. C_{pk} is a high-amplitude torque generates by the gas combustion. Due to its large magnitude, it is not suitable to be countered by the active filter. C_{ik} is a low-amplitude torque that cause vibrations in the vehicle. These vibrations ought to be eliminated by proper opposite torque.

Figure 4 presents a synoptic of the control strategy :

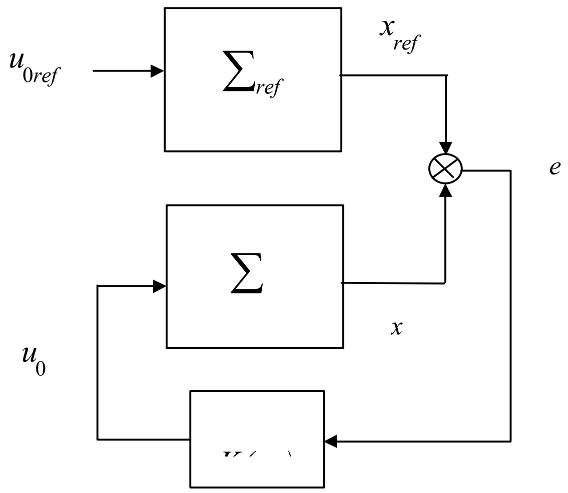

Figure 4: Control structure

Here $K(\omega)$ denotes the controller to be designed. The state space representation of the system $\sum$ is derived from (4) :

$$
\begin{aligned}
\dot{x} &= A(\omega)x + B_w w + B_u u_0 \\
e &= C_e x + D_{ew} w + D_{eu} u_0 \\
y &= C_y x + D_{yu} u_0
\end{aligned} \tag{7}
$$

where $x^T = [i_d, i_q, v_d, v_q, i_{dl}, i_{ql}]$ is the state vector, $u_0^T = [u_{d0}, u_{q0}]$ is the input vector, $w^T = [x_{ref}, \omega]$ is the vector of exogenous inputs, $x_{ref}^T = [i_{dref}, i_{qref}, v_{dref}, v_{qref}, i_{dlref}, i_{qlref}]$ is the reference state vector, $e{=}x{-}x_{ref}$ is the vector of controlled outputs and $y{=}x$ is the vector of measured outputs. We have:

$$
B_w = [O_{6\times 6} \quad F] \qquad F' = \begin{bmatrix} 0 & -\dfrac{\lambda p}{L_q} & 0 & 0 & 0 & 0 \end{bmatrix} \quad C_e = I_{6\times 6} \quad D_{ew} = [-I_{6\times 6} \quad O_{6\times 1}]
$$

$$
D_{eu} = O_{6\times 2} \quad C_y = I_{6\times 6} \quad D_{yu} = O_{6\times 2}
$$

Matrix A depends affinely on parameter ω. It is of the form $A = A_0 + A_1\omega$ where A_0 and A_1 are known fixed matrices. A belongs to polytope type domain D_p given by:

$$
D_p = \left\{ A : A(\alpha) = \sum_{i=1}^{N=2} \alpha_i \overline{A}_i \ , \ \sum_{i=1}^{N=2} \alpha_i = 1,\ \alpha_i \geq 0 \ \forall i \right\} \tag{8}
$$

We assume that each coefficient α_i is real-time computable and it ranges between known extremal values $\alpha_i \in [\underline{\alpha}_i, \overline{\alpha}_i]$, and the variation rate of each coefficient α_i is limited by known upper and lower bounds, that is $|\dot{\alpha}_i| \leq d_i$.

The reference model $\sum_{ref}$ is defined as

$$
\begin{aligned}
i_{dref} &= 0 \\
i_{qref} &= -\frac{2}{3\lambda p} \sum_{k=0}^{3} C_{ik} \\
v_{dref} &= -L_q p \omega i_{qref} \\
v_{qref} &= L_q \dot{i}_{qref} + R i_{qref} + \lambda p \omega \\
i_{dlref} &= C_a \dot{v}_{dref} - \omega C_a v_{qref} \\
i_{qlref} &= C_a \dot{v}_{qref} + \omega C_a v_{dref} + i_{qref}
\end{aligned} \tag{9}
$$

Note that the reference model has the same structure than the system and can be expressed as

$$\dot{x}_{ref} = A(\omega)x_{ref} + B_u u_{ref} + \begin{bmatrix} 0 \\ -\frac{\lambda p}{L_q} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \omega \tag{10}$$

u_{ref} is a fictive input. It corresponds to the optimal input to apply on the system to achieve $x = x_{ref}$. We have :

$$u = K(\omega)\left(x - x_{ref}\right) = u_0 - u_{ref} \tag{11}$$

Denoting by $T_\infty(s)$ and $T_2(s)$ the closed-loop transfer functions from $\begin{pmatrix} x_{ref} \\ \omega \end{pmatrix}$ to e and y respectively, our goal is to design a state-feedback law $u_0 = K(\omega)e$ that maintains the RMS gain (H_∞ norm) of $T_\infty(s)$ below some prescribed value $\gamma_0>0$, maintains the H_2 norm of $T_2(s)$ (LQG cost) below some prescribed value $\upsilon_0>0$ and minimizes an H_2/H_∞ trade-off criterion of the form $a\|T_\infty\|_\infty^2 + b\|T_2\|_2^2$.

Consider the plant defined by (7) and the control law (11). The closed-loop system is stable and $\|T_\infty\|_\infty < \gamma_0$, $\|T_2\| < \gamma_0$ if and only if there exist 2 symmetric positive definite matrices $X_i = X_i^T > 0$, 2 matrices $L_i \in \Re^{n_u \times n}$, as well as a matrix $H \in \Re^{(n+n_u)\times(n+n_w)}$, 2 symmetric positive definite matrices $Q_i \in \Re^{n_u \times n_y}$, $U \in \Re^{(n+n_u)\times(2n+n_u+n_w)}$ and $V \in \Re^{(n+n_u)\times(2n+n_u+n_y)}$ such that $\forall i = 1,2$:

$$\begin{bmatrix} \sum_{k=1}^{2} d_k X_k & X_i & L_i^T & B_w & 0 \\ X_i & 0 & 0 & 0 & 0 \\ L_i & 0 & 0 & 0 & 0 \\ B_w^T & 0 & 0 & -\gamma_0 I & D_{ew}^T \\ 0 & 0 & 0 & D_{ew} & -\gamma_0 I \end{bmatrix} + Sym\left\{ \begin{bmatrix} A_i & B_u \\ -I & 0 \\ 0 & -I \\ 0 & 0 \\ C_e & 0 \end{bmatrix} H \right\} < 0$$

$$\begin{bmatrix} \sum_{k=1}^{2} d_k X_k & X_i & L_i^T & B_w \\ X_i & 0 & 0 & 0 \\ L_i & 0 & 0 & 0 \\ B_w^T & 0 & 0 & -I \end{bmatrix} + Sym\left\{ \begin{bmatrix} A_i & B_u \\ -I & 0 \\ 0 & -I \\ 0 & 0 \end{bmatrix} U \right\} < 0$$

$$\begin{bmatrix} X_i & X_i & L_i^T & 0 \\ X_i & 0 & 0 & 0 \\ L_i & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_i \end{bmatrix} + Sym\left\{ \begin{bmatrix} 0 & 0 \\ I & 0 \\ 0 & I \\ -C_y & 0 \end{bmatrix} V \right\} > 0$$

$$trace\{Q_i\} < \gamma_0^2$$

The state feedback matrix is given by

$$K(\alpha) = L(\alpha) X^{-1}(\alpha) \tag{12}$$

with

$$L(\alpha) = \sum_{i=1}^{2} \alpha_i L_i \quad \text{and} \quad X(\alpha) = \sum_{i=1}^{2} \alpha_i X_i$$

4. Numerical results

In the following section, the control law is applied to the simulation model. The parameters of the combustion engine are based on a 90 kW Diesel engine. The reference speed is fixed to 1900 rpm. The currently injected fuel mass determines the energy release during the next stroke of the engine. The parameters of the electrical actuator are based on a 40 kW permanent magnetic synchronous machine.

The control laws is applied to the combustion engine. The speed ω ranges between known extremal values $\omega \in [50\ rd/s\ , 400\ rd/s]$, and the variation rate is limited by known upper and lower bounds $\left[-50\ rd.s^{-2}, 50\ rd.s^{-2}\right]$.

The design gives the stable control

$$L_1 = \begin{pmatrix} -0.746 & 0.0016 & 1.034 & -0.0062 & -1.57 & 0.0056 \\ 0.0127 & -0.73 & -0.0087 & 1 & 0.0052 & -1.54 \end{pmatrix}$$

$$L_2 = \begin{pmatrix} -0.75 & 0.0087 & 0.984 & -0.00196 & -1.56 & 0.0032 \\ 0.016 & -0.73 & -0.73 & -0.0037 & 0.0036 & -1.54 \end{pmatrix}$$

$$X_1 = \begin{pmatrix} 0.55 & 0.0007 & -0.74 & -0.00003 & 0.53 & 0.0005 \\ 0.0007 & 0.55 & 0.00016 & -0.744 & 0.00048 & 0.53 \\ -0.74 & 0.00016 & 1.04 & -0.000017 & 0.75 & -0.0013 \\ -0.00003 & -0.744 & -0.000017 & 1.04 & 0.0015 & -0.75 \\ 0.53 & 0.00048 & 0.75 & 0.0015 & 0.552 & 0.0022 \\ 0.0005 & 0.53 & -0.0013 & -0.75 & 0.0022 & 0.55 \end{pmatrix}$$

$$X_2 = \begin{pmatrix} 0.59 & 0.0035 & -0.74 & 0.000035 & 0.57 & 0.0012 \\ 0.0035 & 0.59 & -0.00083 & -0.74 & 0.0027 & 0.56 \\ -0.74 & -0.00083 & 0.97 & 0.0002 & 0.75 & -0.001 \\ 0.000035 & -0.74 & 0.0002 & 0.97 & 0.57 & -0.75 \\ 0.57 & 0.0027 & 0.75 & 0.57 & 0.58 & 0.00094 \\ 0.0012 & 0.56 & -0.001 & -0.75 & 0.00094 & 0.58 \end{pmatrix}$$

Coefficients α_i are given by :

$$\alpha_1 = \omega(t) - \frac{\omega_{max}}{\omega_{max} - \omega_{min}} \qquad \text{and} \qquad \alpha_2 = 1 - \alpha_1$$

and the state feedback matrix is given by (12) where $L(\alpha) = \alpha_1 L_1 + (1-\alpha_1)L_2$ and $X(\alpha) = \alpha_1 X_1 + (1-\alpha_1)X_2$.

The electrical actuator generates a zero mean inverse torque to compensate the torque ripples of the engine (fig. 5).

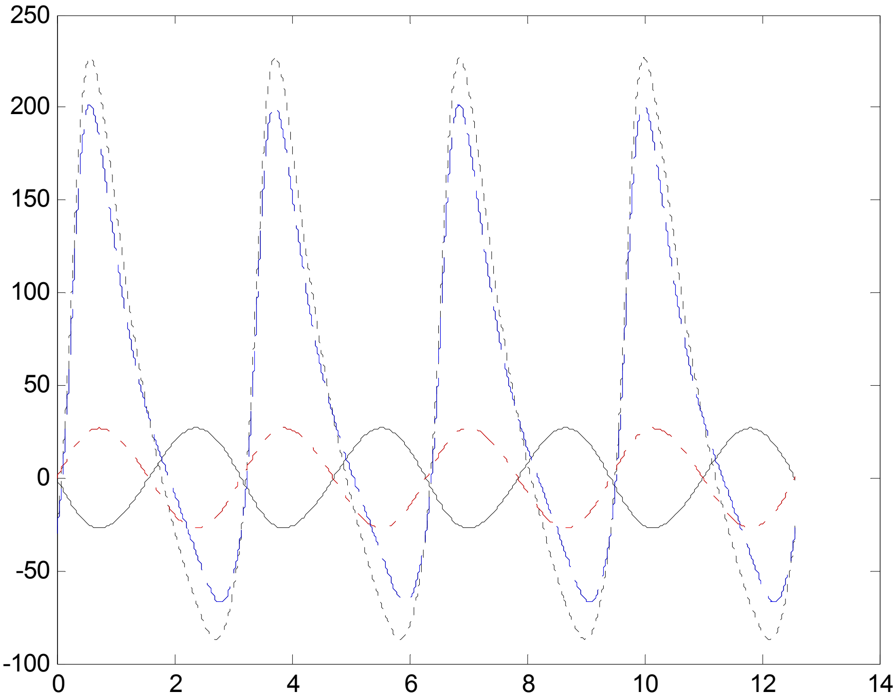

Fig. 5: Inertial torque (black solid lines), counter torque (red dashdot lines), resulting torque (black dotted lines), initial torque (blue dashed lines) in N.m for $\omega = 200 rad/s$, versus θ.

5. Conclusion

In this paper, we present a novel control strategy of torque ripple on hybrid vehicle. A zero mean inverse torque is generated by an AC machine to compensate the ripples. The control law is based on a state feedback, which is well suited for electrical drive. A LMI framework is used to synthesis the controller. Simulation results show the interest of this approach.

6. References

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