

# Energy-efficient traction supply system of modern trams

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*Abstract* – Modern tram can be energy-efficient due to recuperative braking. A moving tram has kinetic energy, which can be transformed into electric energy during *recuperative* braking. It can be fed back into contact system free of charge. Therefore decrease in energy consumption is possible.

However present supply conditions limit energy-efficiency of expensive modern tram.

Reasons for modernization of existing traction supply systems in Katowice are presented. It would improve energy-efficiency, reduce voltage drops and energy losses. Since classical investment (substations, feeder cables and cross-sections) requires huge financial outlay, in the future other solution can be adopted (energy accumulator).

Therefore presented technical aspects describe necessary network adaptation and the method of evaluating its real influence on supply system. This is useful to the companies negotiating the purchase of modern trams to evaluate real advantages of investing additional financial resources indispensable for buying of recuperative braking trams.

## Introduction

Nowadays many tram operators are modernising their rolling stock, which make possible recuperation of braking energy into the contact system. Modern trams differ from currently operating trams (mostly 105N type) - power electronics converters are used in traction motors supply systems. These converters make possible:

- 1. limiting energy losses during start-up,**
- 2. energy recuperation during braking and downhill run.**

Modern tram can be energy-efficient due to recuperative braking. However present supply conditions limit energy-efficiency of expensive modern tram.

## Kinetic energy transformations during tram braking

A moving, modern tram has kinetic energy, which can be transformed into electric energy during braking. Energy of a moving tram during braking is expended in following ways:

- I) to overcome resistance to traction (which is present during braking as well as during running);
- II) to cover power losses in the drive system transforming kinetic energy into electric energy;

- III) to supply auxiliaries of braking tram (also for heating in winter);
- IV) whenever possible<sup>1</sup>, this energy is **recuperated** into contact system and used up by other trams provided that:
  - a) demand for energy occurs at a given time instant and
  - b) the contact system is not overloaded.
- V) excess energy, which cannot be recuperated into the contact system, is **irrevocably lost** – transformed into **heat** in braking resistors.

Given feeder cable load measurements, it is difficult to calculate how much excess energy is lost in braking resistors, when it is not recuperated into the contact system, just because contact system is not adapted to the recuperation demands.

Introducing modern trams requires additional investments, because the supply system must be modernised. The control system of the new tram does not tolerate significant voltage drops in contact system.

Reliable operation of modern trams, which puts greater demands on supply system operation, and its effective recuperative braking requires a more powerful supply system, i.e. it is necessary to erect new traction substations and to lay out new feeder and return cables with increased cross-sections, in order to limit voltage drops in contact system.

However, modernising the supply system alone, does not guarantee that recuperative systems of modern trams will enable complete recovery of braking energy into the power network, which seems to be the advertising slogan of manufacturers of new and costly trams.

Increasing energy consumption effectiveness by recovering trams' braking energy calls for following conditions to be met simultaneously:

- I) modernising of operating trams and purchase of new trams (equipped with recuperative braking circuits)
- II) reconstructing present-day supply systems, which should make possible:
  - 1) operating of new trams, which require improved electric energy quality<sup>2</sup> (voltage drops below 400V are inadmissible in case of 116Nd trams)
  - 2) transferring recuperated energy to another tram
- III) effective utilising of available braking energy (it must be recovered during braking) by:
  - 1) construction of systems<sup>3</sup> making possible transfer and sale of recovered energy to the power network,
  - 2) developing trams' schedule in such a way, that braking energy of one tram can be used up by other trams operating at maximum power (during start-up). Japanese Railways operate like this, due to fully automated synchronisation of train runs on chosen routes<sup>4</sup>.

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<sup>1</sup> in 116Nd and modernised 105Na trams

<sup>2</sup> Quality of energy is closely related to supply system operating conditions. The limit values imposed on 600V d.c. tram systems are:

$U_{min1} = 400V$  (minimum continuous voltage;  $(600-33.3\%*600)V$ ,

$U_{max1} = 720V$  (maximum continuous voltage;  $(600+20\%*600)V$ , 105Na trams have been adapted to this voltage

$U_{max2} = 770V$  (maximum voltage of 5-min. duration)

$U_{max3} = 1269 V$  – overvoltage lasting no more than 20ms [11]

<sup>3</sup> **technical** condition is that these systems transform d.c. current into a.c. current of suitable quality, and **economical and legal** condition is that the power company will be prepared to buy a momentary energy surplus from the traction system

<sup>4</sup> rail tracks run independently of car (road) traffic

However, random (stochastic) type of tram operation in the city, where tram tracks often cross car roads, practically precludes full utilisation of braking energy by adaptation of tram schedule so that start-ups and brakings were synchronised<sup>5</sup>.

If allowable voltages are exceeded or when recovered energy cannot be consumed by other trams (in particular, by starting ones), the braking energy must be fed back into power network with the help of inverters or else be dissipated in braking tram's resistors.

### **Analysis of factors limiting the use of tram braking energy**

The energy consumption analyses has been done for "Park Kościuszki" substation, which supplies contact system of tram lines #6 and #16 for route from "Pętla Brynów" to "Plac Miarki" in Katowice. This section of the route is characterised by considerable grades, where trams must carry out braking.

This chapter presents an analytical evaluation of theoretical value of current  $I_B$  generated by tram's drive system during braking. However, it cannot be assumed that the whole of this current is recuperated at the very instant of its generation (some excess braking energy will be dissipated in braking resistors).

The basic factors determining the possibilities of use of braking energy in the supply area of a given traction substation are electric energy demand and traction contact system's low resistance in current  $I_B$  path ( $I_B$  current is generated by drive system of the braking tram).

The maximum value of current  $I_B$  is limited by  $ULB_{max}$

- $ULB_{max}$  - allowable maximum contact line voltage during braking, estimated on the basis of compared 116Nd tram voltage course during braking, when the d.c. transistor chopper is operating in the braking resistor circuit [chopper is switched on by braking voltage] - 720 V),

and voltage at d.c. bus-bars in traction substation  $U_{sub}$ . Current  $I_H$  must, under certain supply conditions, flow into other trams supplied from:

- a) same section of the contact system (Fig. 1),
- b) others sections of traction substation supply area<sup>6</sup>(Fig. 2).

### **Determination of relationship between maximum values of tram recuperative braking current and tram supply system parameters**

The maximum values of braking currents have been calculated for trams' operating conditions corresponding to diagrams shown in figs.: Fig. 1-Fig. 5.

### **Simplifications of the equivalent scheme used in modelling of traction drives supply system**

The basic scheme used in simulation of dynamics of traction drives, where electrical quantities change quickly (voltage or overvoltage rise rates are high), should take into account inductance of the motor armature as well as that of the contact system<sup>7</sup>.

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<sup>5</sup> of trams recuperating braking energy

<sup>6</sup> During measurements conducted in February 2002, current measurement circuit for different feeder cables of "Park Kościuszki" traction substation was established. It has made possible analysis of case b).

<sup>7</sup> cf. MIERZEJEWSKI L.: SZELĄG A., GAŁUSZEWSKI M.: *System zasilania trakcji elektrycznej prądu stałego*, Wydawnictwa Politechniki Warszawskiej, Warszawa 1989 (D.c. *traction supply system*. In Polish)

However, in case of calculations carried out here, concerned with such electrical and mechanical quantities as equivalent current, torque, acceleration, speed and energy consumption, the mathematical model shown in the equivalent scheme given below is accurate enough (the inductances are not considered in this model).

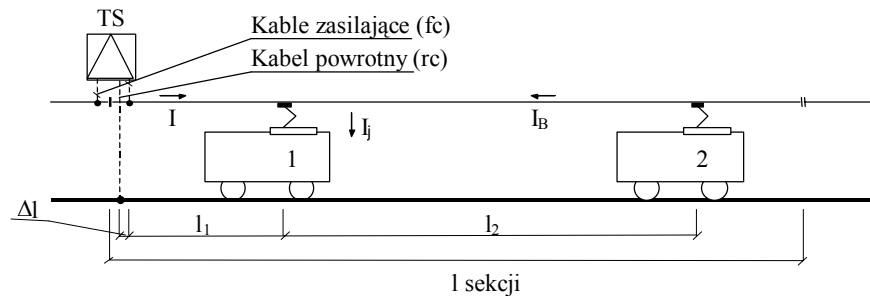


Fig. 1 Connection diagram of trams running over a chosen section of contact system. The system is supplied from traction substation (radial network). 1 – tram consuming energy, supplied from contact system ( $I_j$ ,  $U_{Lj}$  - Fig. 3), 2 – braking tram, generating energy ( $I_B$ ,  $U_{LB}$  - Fig. 3), TS =traction substation ]

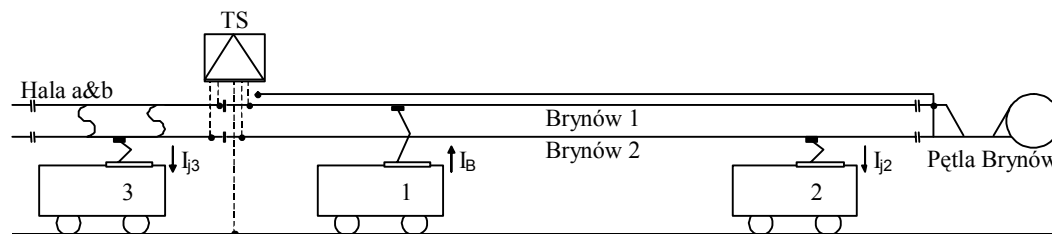


Fig. 2 Trams connected into contact system supplied from traction substation

The following symbols have been used in figures and equations:

- $U_{sub}$  - voltage at d.c. bus-bars in traction substation ;
- $U_{LB}$  - tram braking voltage – contact system voltage at a point, where current  $I_B$  from braking tram flows back into contact system, see  $U_{LBmax}$ ;
- $U_{Lj}$  - contact system voltage at a point, where current  $I_j$  is collected by tram No.1;
- $R_{fc}$  - resistance of a feeder, i.e. a cable connecting the substation (positive bus bar) to the contact line;
- $R_{rc}$  - resistance of a return cable (or cables), connecting the substation (negative bus-bar) to the tram rails;
- $R_c$  - resistance of a feeder and a return cable (cable type- YAKY625mm<sup>2</sup>);
- $R_{sj1}$  - resistance of the contact system between the feeder's point of connection with the contact system and the current collecting site of tram No.1;
- $R_{rail.1}$  - rails' resistance (relating to the contact system section of  $R_{sj1}$  resistance);
- $R_{sj2}$  - contact system resistance between trams No.1 and 2;
- $R_{rail.2}$  - rails' resistance between trams No.1 and 2;
- $R_1, R_2$  - resistances present in braking current path;
- $I_j$  - current consumed by tram;
- $I_B$  - current flowing from tram back into contact system during recuperative braking;
- $I$  - current flowing from substation to contact system section in question;

It has been assumed throughout that tram driver makes decisions about current  $I_j$  value.

Traction substation d.c. bus-bar voltage is defined by the equation:

$$\bullet \quad U_{sub} = U_0 - \Delta U_{sub} = U_0 - I_{sub} \cdot R_{eq} \quad [V] \quad (1)$$

where:

- $U_0$  - traction substation bus-bar no-load voltage;
- $I_{sub}$  - load current of the rectifier sets; it is the sum of feeder cables currents;
- $R_{eq}$  - equivalent resistance of a substation (equivalent resistance of two rectifier sets),
- $\Delta U_{sub}$  - sum of voltage drops, in d.c. voltage terms, depending on the reactance of the network supplying traction substation, rectifier transformers, transformer resistance and voltage drops across rectifier switches.

This equation determines the current-voltage characteristic of the traction substation rectifier sets; the current varies from no-load value up to several times its rated value, while the overlap angle  $\gamma$  changes from 0 to 60 deg (el.)

$U_0$  and  $R_{eq}$  have been calculated on the basis of regression analysis of  $U_{sub}$  measurements and corresponding load current values of two rectifier sets of “Park Kościuszki” substation - $I_{sub}$ .

The following values of parameters have been obtained for a line approximating the current-voltage characteristic of traction substation  $U_{sub}=f(I_{sub})$ :

$$R_{eq}=0,0255 \Omega \quad (2)$$

$$U_0=717,19 \quad (3)$$

$$U_{sub} = -R_{eq} * I_{sub} + U_0 = -0,0255 * I_{sub} + 717,19 \quad (4)$$

Keeping in mind that the rectifier sets load current  $I_{sub}$  depends on other factors besides load current  $I$  (this is current flowing into tram supply section in question), it has been assumed that d.c. bus-bar traction substation voltage  $U_{sub}$  is constant (within the limits set by actual operating conditions).

Resistance of elements of the contact system  $R_{s1}$  and  $R_{s2}$  is the sum of the resistances of overhead contact line (catenary) and rails' resistances (Fig. 3, Fig. 4).

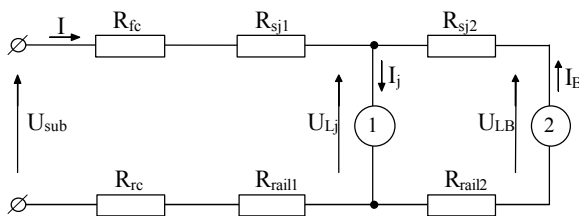


Fig. 3 Diagram of trams connected into contact system  $R_{fc}$ ,  $R_{rc}$  – cable resistance

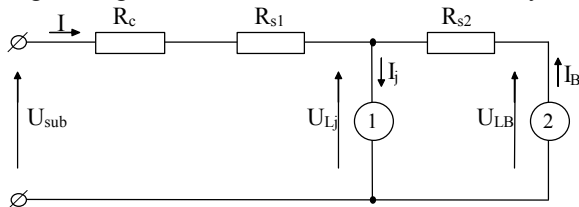


Fig. 4 Simplified diagram of trams connected into contact system. Tram no.1 collects energy.  $R_c$  – cable resistance, other symbols as in Fig. 1.

$$R_{s1} = R_{sj1} + R_{rail.1} [\Omega] \quad (5)$$

$$R_{s2} = R_{sj2} + R_{rail.2} [\Omega] \quad (6)$$

$$R_{s1} = l_1 * r_{s1} [\Omega] \quad (7)$$

– for a homogenous contact system (where the overhead wire and rail materials and dimensions do not vary throughout):

$$rs_1=rs_2=rs[\Omega/\text{km}] \quad (8)$$

$rs$  - sum of specific resistances of overhead contact line  $rs_j$  and rails  $r_{\text{rail}}$  [ $\Omega/\text{km}$ ]  
 $rs=rs_j+r_{\text{rail}}$  (9)

Current consumed by tram No.1 (supplied from contact system)

$$I_j=I+I_B \rightarrow I=I_j-I_B \text{ [A]} \quad (10)$$

The maximum value of current  $I_B$  generated by a braking tram has been determined on the basis of following equations:

$$U_{Lj}=U_{\text{sub}}-(R_c+R_{s1}) \cdot I=U_{LB}-R_{s2} \cdot I_B \text{ [V]} \quad (11)$$

Taking into account eq. (10), we obtain:

$$I_B=[U_{LB}-U_{\text{sub}}+I_j \cdot (R_c+R_{s1})]/(R_c+R_{s1}+R_{s2}) \text{ [A]} \quad (12)$$

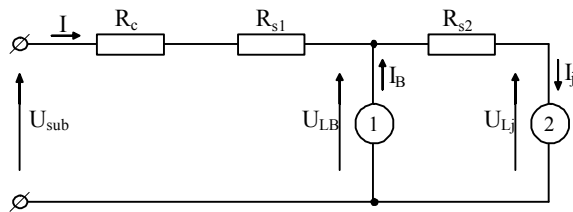


Fig. 5 Simplified diagram of trams connected into contact system. Tram No.1 generates braking energy

For a scheme shown in Fig. 5, where braking tram is closer to the traction substation than the other tram, and assuming that tram No.1 braking power is equal to  $P_B^8$ , and taking into account Eq. (10), the following equations are obtained:

$$U_{Lj}=U_{LB}-I_j R_{s2} \quad (13)$$

$$U_{LB}=U_{\text{sub}}-I(R_c+R_{s1}) \quad (14)$$

$$P_B=I_B U_{LB} \quad (15)$$

$$aI_B^2+bI_B+c=0 \quad (16)$$

$$a=R_{s2} \quad (17)$$

$$b=[I(R_c+R_{s1}+R_{s2})-I_j R_{s2}-U_{\text{sub}}] \quad (18)$$

$$c=P_B \quad (19)$$

However, when the recuperative braking current  $I_B$  flows into traction substation positive d.c. bus-bar and subsequently into other supply sections of the contact system (Fig. 2) and when  $I_B < I_{j2}$ , the relationships between supply and load parameters may be set down in the form of equations calculated on the basis of simplified diagram of the system (Fig. 6)

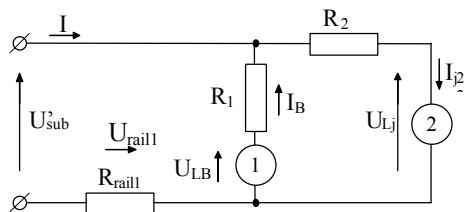


Fig. 6. Simplified diagram of trams No.1 and No.2 connected into contact system shown in Fig. 2

$$U'_{\text{sub}}=U_0-I_{\text{sub}}(R_z+R_{rc}) \quad (20)$$

<sup>8</sup> this value is set by tram driver

$$R1=R_{fc}+R_{sj1} \quad (21)$$

$$R2=R_{fc}+R_{s2} \quad (22)$$

$$U'_{sub}=ULB-IB \cdot R1+(I_{j2}-IB)R_{rail.1} \quad (23)$$

$$ULB-IB \cdot R1=U_{sub}-\Delta U_{rail.1}=I_{j2} \cdot R_{zj2} \quad (24)$$

$$\Delta U_{rail.1}=I \cdot R_{rail1} \quad (25)$$

$$R_{zj2}=(U_{sub}-\Delta U_{rail1})/I_{j2} \quad (26)$$

$$ULB=PB/IB \quad (27)$$

$$R1IB^{(2)}+I_{j2}R_{zj2}IB-PB=0 \quad (28)$$

### Comparison of maximum braking current: calculated values (computed on the basis of estimated supply system parameters) and real values (measured in feeder cables)

Examples of calculations which follow and measurement results relate to the supply system consisting of a traction substation feeding two-track tram line (without any side branches) via contact system consisting of several sections. Such a layout corresponds to the contact system of tram lines #6 and #16, from "Pełta Brynów" to "Plac Miarki" in Katowice, supplied from "Park Kościuszki" substation (Fig. 2). Modern 116Nd trams, with recuperative braking, have been put into operation on line #6 in 2001.

Analysing the schedules of these two tram lines over the given route it is seen that number of trams or tramsets supplied from one particular section of the contact system does not exceed two. Therefore, in the diagrams of tram supply circuits (diagram corresponds to supply system of a given supply section), two trams or tramsets operating have been taken into account: one is supplied from the contact system (current  $I_j$ ) and the other is braking and supplies the contact system with  $I_B$  current.

Simulation results of 116Nd trams operating in the supply area of "Park Kościuszki" substation show that the maximum currents generated by trams' drive systems are contained within the limits  $I_{Bmax} = (500-750)A$ .

The recorded measurement results of load currents of substation's different feeders show that under actual operating conditions braking currents  $I_B$  of 116Nd trams are not greater than 200A (Fig. 7-Fig. 10), with the exception of instance illustrated in Fig. 10, where  $I_B$  is equal to 366A. Diagrams show feeder cables' currents (currents flowing from d.c. positive bus-bar of "Park Kościuszki" traction substation to differing contact system sections), when trams just start to operate in the early morning.

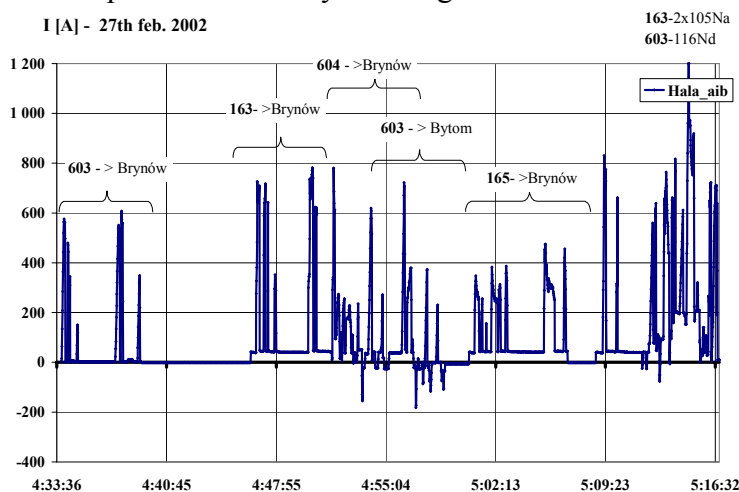


Fig. 7 Feeder load current courses, for feeders "Hala a" and "Hala b". Trams (No. 603, 163, 604, 165) are running in the same direction, towards Brynów. Tram No. 603 is running towards centre of Katowice (and next to Bytom)

The first tram running on line#6 (No.603, 116Nd tram) entered “Park Kościuszki” substation area at 4:33 (Fig. 7), after crossing section isolator uphill and away from “Plac Miarki” stop. Since there were no possible energy recipients within the area, the recuperative current had to be limited to the value set by load current of tram’s own auxiliaries

A similar load current course is shown in Fig. 8 (tram No.603 running over “Brynów 2” section); over a slope of significant gradient and with imposed speed limit of  $v_{max} = 60\text{km/h}$  drive motors operated at power close to motors’ rated power, current  $I_j$  rising up to 750A.

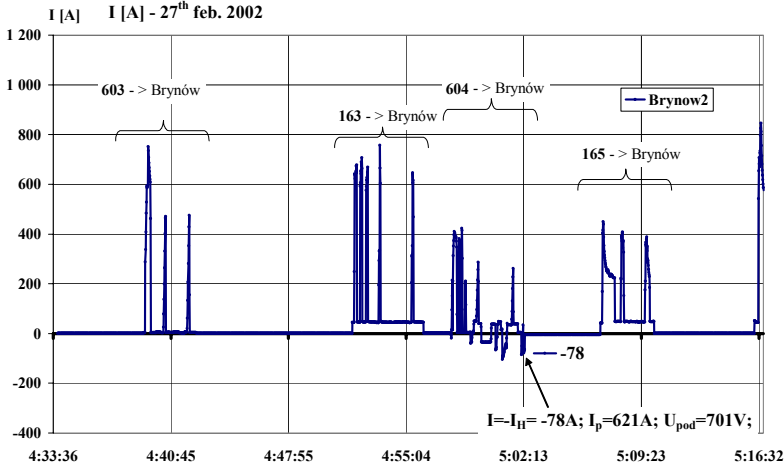


Fig. 8 Feeder load current course (feeder "Brynów 2"). Trams (No. 603, 163, 604, 165) are running towards Brynów.

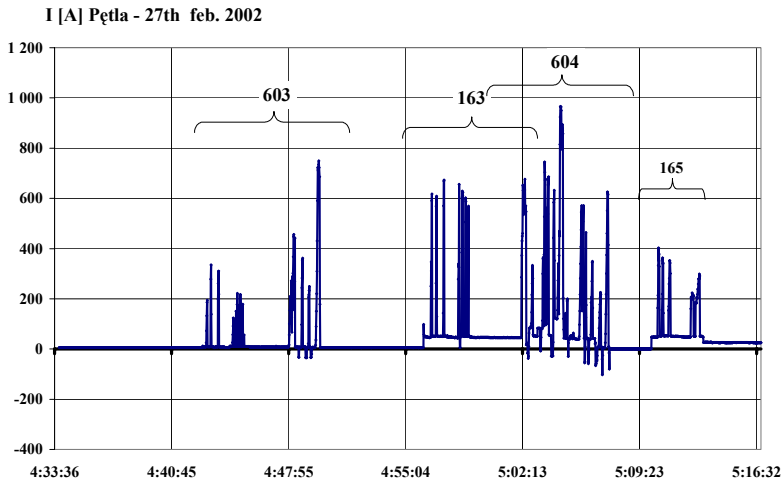


Fig. 9 Feeder load current course (feeder "Pętla Brynów "). Trams (No. 603, 163, 604, 165)

Only when tram in question (No.603) entered “Pętla Brynów” (at 4:48hrs.), recuperative braking current could be observed. The braking current was supplied to next tram entering “Park Kościuszki” supply area (2x105Na tramset No.163, line#16) – Fig. 7.  $I_B$  current was consumed by No.163 tram’s auxiliaries.



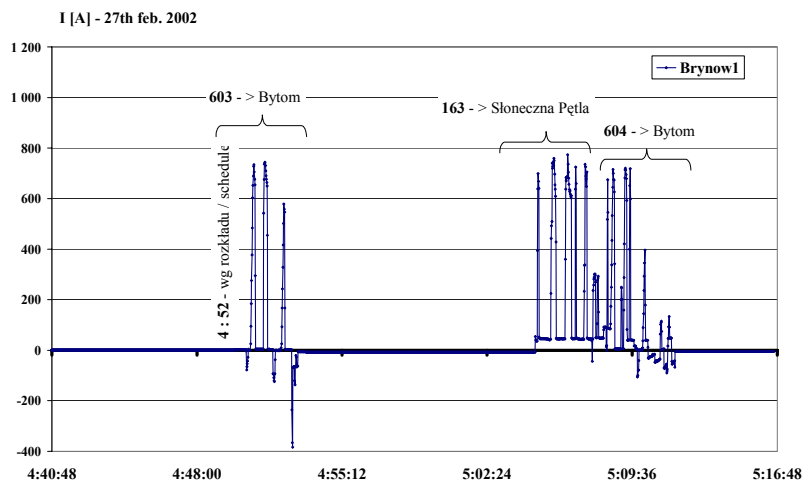


Fig. 10 Feeder load current (feeder "Brynów 1"). Trams (No. 603, 163, 604) are running in the same direction, towards centre of Katowice (and next to Bytom or Sloneczna Petla)

$I_B$  current maximum value (366A) occurs at time instant, when tram No.603 brakes (running to "Park Kościuszki" stop directly in front of the substation) – Fig. 2. At this route point feeders of "Brynów1", "Brynów2" and "Hala a & b" sections are connected into the contact system; system is subdivided into sections by section isolators. Current path resistance in the is at its minimum, and maximum load currents of trams No.163 (Fig. 8,  $I_{j2} = 669A$ ) and No.604 (Fig. 7,  $I_{j3} = 170A$ ) occur at time instant, when tram No. 603 brakes (at 4:52:44 hrs.).

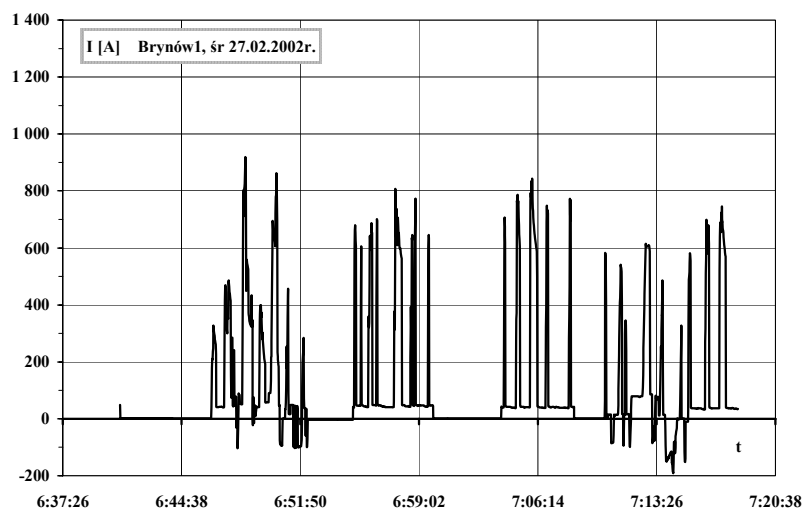


Fig. 11 Feeder load current course, recorded on 27<sup>th</sup> Feb., 2002 in "Park Kościuszki" substation from 6:37 to 7:16. This feeder supplies "Brynów 1" section of the contact system

Fig. 11 shows feeder load current courses. This particular feeder supplies "Brynów 1" section, connecting "Petla Brynów" with "Park Kościuszki" stop. Trams run over this route section in just under 7 minutes (there are two intermediate tram stops and two road crossings with traffic lights). Two central "crests" correspond to load currents caused by two trams running over the section (two successive tramsets 2x105Na). The first "crest" relates to 116Nd tram run, and the last "crest" corresponds to the sum of overlapping load currents of 116Nd and 2x105Na trams.

**Example No.1. Calculation of maximum braking current of tram No.604; tram was leaving contact system section "Brynów 2" (Fig. 8; 5:02:13 hrs)**

The specific resistance of catenary of C70-1c type, consisting of messenger cable C70 ( $r_j=0.285 \Omega/\text{km}$ ) and contact wire  $D_{jp}100$  ( $r_{pj} = 0.1984 \Omega/\text{km}$  – estimated value, 10 per cent wear assumed)

$$r_{sj}=r_{pj} \cdot r_l / (r_{pj} + r_l) = 0,117 \text{ } \Omega/\text{km} \quad (29)$$

S42 rails specific resistance (two tracks):

$$r_{rail}=0,25 \cdot 1,5 / 49 = 0,0077 \text{ } \Omega/\text{km} \quad (30)$$

Contact line specific resistance

$$r_s=r_{sj}+r_{rail}=0,1247 \text{ } \Omega/\text{km} \quad (31)$$

Specific resistance (per 1 km) of YAKY625mm<sup>2</sup> cable

$$r_{1c}=1000 / (\gamma \cdot s) = 1000 / (34 \cdot 625) = 0,0471 \text{ } \Omega/\text{km} \quad (32)$$

Total resistance of feeder and return cables ( $R_{fc}$  and  $R_{rc}$ , respectively), YAKY625mm<sup>2</sup>,  $\Sigma l_c = 0.3 \text{ km}$

$$R_c=r_{1c} \cdot \Sigma l_c = 0,0471 \cdot 0,3 = 0,0141 \text{ } \Omega \quad (33)$$

For  $I_j=0$ ;  $R_{s1}=0$ ;  $I = -I_B$  (Fig. 4),

Resistance of „Brynów2” contact system section (1.660 km long ) is equal to

$$R_{s2}=R_s \cdot l_s = 0,1247 \cdot 1,66 = 0,2070 \text{ } \Omega \quad (34)$$

$$R_c + R_{s2} = 0,2211 \text{ } \Omega \quad (35)$$

at:

$U_{sub}$  - substation bus-bar voltage at  $I_{sub}=621 \text{ A}$  (value recorded at 5:02:13; 701V),

$$I_B = I_{Bmax} = [U_{LBmax} - U_{sub} + I_j(R_c + R_{s1})] / (R_c + R_{s1} + R_{s2}) = (720 - 701 + 0) / 0,2211 = 85,9 \text{ } \text{A} \quad (36)$$

„Brynów2” feeder current recorded at 5:02:13 and corresponding to 116Nd tram braking current (tram No.604, line#6) was equal to  $I_B = 78.1 \text{ A}$  (Fig. 8).

**Example No.2. Calculation of maximum braking current of tram No.603; tram was approaching tram stop "Park Kościuszki"; (Fig. 10; 4:52:44 hrs)**

At given time instant trams were spaced along the route as in Fig.2. Recorded current values were:  $I_{j2}=669 \text{ A}$ ;  $I_{j3}=170 \text{ A}$ .

Tram No.1 (No. 603) was located  $l_1 \approx 100 \text{ m}$  away from “Brynów 1” section feeder connection point, and tram No.2 (No.163) was located  $l_2 \approx 570 \text{ m}$  from tram No.1.

The recorded load parameters values for circuit shown in Fig.2 were equal to:  $U_{sub} = 705 \text{ V}$ ,  $I_{sub} = 473 \text{ A}$ ,  $I_{j2} = 669 \text{ A}$ ,  $I_{j3} = 170 \text{ A}$ ,  $I_B = 366 \text{ A}$ .

Equivalent resistance and rectifier sets no-load voltage (1) are equal to:  $R_{eq} = 0,0255 \text{ } \Omega$ ,  $U_0 = 717 \text{ V}$ .

Fig. 2 shows how the different trams are located with respect to the feeder; simplified circuit scheme shown in Fig. 6 corresponds to this layout. It has been assumed that substation voltage is equal to potential difference between d.c. positive busbar (“+”) and tram rails at a point, where the return cable is connected. Resistance of return cable  $R_{rc} = 0,0071 \text{ } \Omega$  ( $l_{rc} = 0,15 \text{ km}$ ;  $r_{1k} = 0,0471 \text{ } \Omega/\text{km}$  for  $1 \times 625 \text{ mm}^2 \text{ Al}$  cable).

$$R'_{eq} = R_{eq} + R_{rc} = 0,0255 + 0,0071 = 0,0326 \text{ } \Omega \quad (37)$$

$$U_{sub}' = U_0 - I_{sub} R'_{eq} = 717 - 473 \cdot 0,0326 = 701,6 \text{ } \text{V} \quad (38)$$

Braking current can be calculated from Eq.(28), provided  $P_B$  and  $R_{zj2}$  values are known, or Eq. (23) may be used if  $U_{LB}$  value is given:

$$\Delta U_{rail.1} = (I_{j2} - I_B) R_{rail.1} = 0,5 \text{ } \text{V} \quad (39)$$

$$R_{zj2} = (U_{sub} - \Delta U_{rail.1}) / I_{j2} = (U_{sub} - 0,5) / 669 = (701,1) / 669 = 1,048 \text{ } \Omega \quad (40)$$

$$R_1 = R_k + R_{sj1} = r_{1klk} + r_{sj1} = 0,0471 * 0,15 + 0,117 * 0,1 = 0,0188 \text{ } \Omega \quad (41)$$

$$R_2 = R_k + R_{s2} = r_{1klk} + r_{s2l2} = 0,0471 * 0,15 + 0,1247 * 0,57 = 0,0782 \text{ } \Omega \quad (42)$$

$$R_{rail.1} = 11 r_{rail} = 0,1 * 0,0077 \approx 0,0008 \text{ } \Omega \quad (43)$$

To check whether  $I_B$  current has attained its maximum value limited by supply system parameters,  $U_{LB}$  voltage of a braking has been calculated with the help of Eq. (23):

$$U_{LB} = U_{pod} + I_B * R_1 - (I_{j2} - I_B) R_{rail.1} = 701,6 + 366 * 0,0188 - (669 - 366) * 0,0008 = 708,2 \text{ V} \quad (44)$$

In case of circuit shown in Fig. 2 maximum braking current  $I_B$  cannot exceed:

$$I_{Bmax} \leq (I_{j2} + I_{j3}) = 669 + 170 = 839 \text{ A} \quad (45)$$

Taking into account the fact, that  $U_{LB}$  cannot exceed  $U_{LBmax} = 720 \text{ V}$ , maximum braking current has been calculated for this voltage value on the basis of Eqs. (20) and (23):

$$U_{LB} = U_0 - I_{sub} * R_{eq} + I_B R_1 - (I_{j2} - I_B) * R_{rail.1} \quad (46)$$

$$I_{sub} = I_{j2} + I_{j3} - I_B \quad (47)$$

$$I_B = I_{Bmax} = [U_{LBmax} - U_0 + R'_{eq}(I_{j2} + I_{j3}) + R_{rail.1} * I_{j2}] / (R'_{eq} + R_1 + R_{rail.1}) \quad (48)$$

$$I_{Bmax} = (720 - 717 + 0,0326(669 + 170) + 0,0008 * 669) / (0,0326 + 0,0188 + 0,0008) = 592 \text{ A} \quad (49)$$

Results of braking currents calculations set out above show how differing factors such as traction operating parameters, rolling stock characteristics and supply system resistances influence the limits of maximum braking current  $I_B$  and braking power  $P_B$ .

One of the factors limiting use of braking trams energy is the supply system itself (contact lines resistance, rail resistance, feeder cables resistance - in the current path between braking tram and tram consuming braking energy recuperated into the contact system). In order to make full use of recuperative braking energy, correct design of trams' supply system should take into account maximum power  $P_B$ . For instance, example 2 and Eq.(28) demonstrate maximum power to be equal to:

$$P_B = I_B * U_{LB} = 366 * 708,2 = 259201 \text{ W} \quad (50)$$

This power can be calculated on the basis of assumed parameters: acceleration, speed, resistance to traction, tram weight, tram's auxiliaries power and coefficients of efficiency of drive system. For the equivalent resistance of the supply circuit of tram No.2 of Fig.6 (and in accordance with Eq. (28)), braking current is defined as:

$$0,0188(I_B)^2 + 669 * 1,048 * I_B - 259,2 * 10^3 = 0 \quad (51)$$

$$\Delta = 0,5 = 714,9$$

$$I_B = (-701,1 + 714,9) / (2 * 0,0188) = 367 \text{ A}$$

Current value, which has been measured, is equal to 366A.

## PRESENT-DAY CONDITION AND POSSIBLE MODIFICATIONS OF SUPPLY SYSTEM

### Analysis of load courses of a chosen contact line supply section

Section of the contact system located at beginning of tram lines #6/#16 has been chosen as a data source for investigation of load courses. This section includes "Pętla Brynów" (loop - in Brynów) – last stop of lines#6/#16, one-track access to the loop and two-track route section with a stop. The load course of investigated section is characterised by the fact, that starting currents of several trams occur almost simultaneously, hence load current may surge up to 1400 A. Recorded load courses of a chosen feeder are depicted in Fig. 12.

Zmierzony prąd I [A] zasilacza Brynów Pętla - środa 27.02.2002 r.

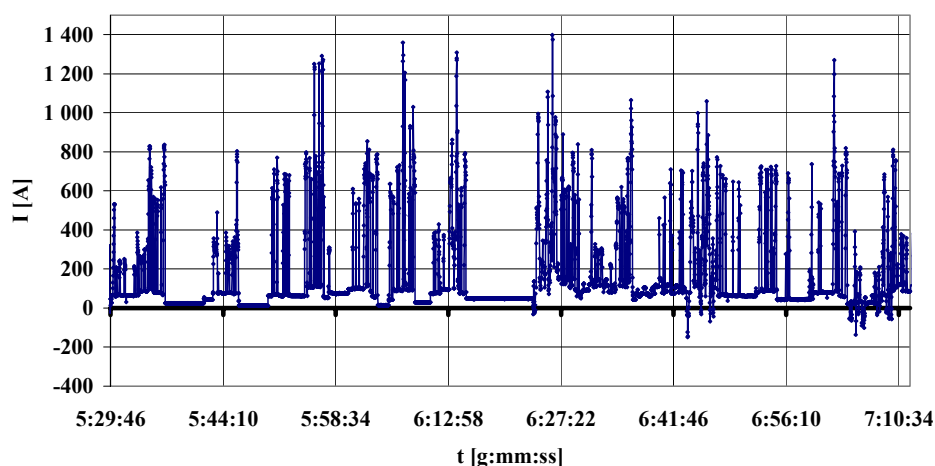


Fig. 12 “Pętla Brynów” feeder load course (27<sup>th</sup> feb. 2002) with instantaneous current as high as 1400 A

Lower range of feeder load relates to currents supplied to auxiliary circuits of trams accessing or leaving the loop or standing there. These currents varied between 20 and 120 A (Fig. 12).

With a maximum “Pętla Brynów” feeder load current of  $I_{max}=1400$  A (Fig. 12) and load current of all rectifier sets equal to the sum of currents of all substation feeders  $I_{sub}=\Sigma I=2000$ A, line voltage  $U_L$  at site, where feeder is connected into the contact system can be calculated in accordance with Eq. (1) as:

$$U_L = U_{sub} - I_{max} * R_{circ.suppl.} = U_0 - I_p * R_{eq} - I_{max} * R_{circ.suppl.} \quad (52)$$

where:

$U_{sub}$ ,  $U_0$ ,  $R_{eq}$  - as in Eq. (1);

#### Calculation of supply system resistance $R_{circ.suppl.}$

Total resistance of feeder cable ( $R_{fc}$ ) and return cable ( $R_{rc}$ ), with cable parameters YAKY625mm<sup>2</sup>, and length  $\Sigma l_c=0,3$  km (cf. Eq. (33)) – is equal to  $R_c=0,0141\Omega$ .

$$R_c = (R_{fc} + R_{rc}) \quad (53)$$

$$R_c = r_{lc} * (l_{fc} + l_{rc}) = 0,0471 * (0,15 + 0,15) = 0,0141 \Omega \quad (54)$$

Additionally:

$R_l$  – resistance of feeder wire connected to “Pętla Brynów” section

$$R_l = 1750 / (34 * 240) = 0,2145 \Omega \quad (55)$$

Rails’ resistance (2 tracks – cf. Eq. (30)) is equal to

$$R_{rail} = 0,0077 * 1,8 = 0,0139 \Omega \quad (56)$$

and taking into account specific S42-type rails’ resistance (2 tracks)  $r_{rail}=0,0077 \Omega/km$ ,  $R_{l\_BP}$  has also been calculated:

$R_{l\_BP}$ . – contact system section resistance (contact line and rails); average distance is taken into account – i.e. distance between the feeder's point of connection with the “Pętla Brynów” contact system section and the current input site of tram running within this section

$$R_{l\_BP} = (0,5 r_{sj} + r_{rail}) * l_{BP} = (0,5 * 0,177 + 0,0077) * 0,25 = 0,0241 \Omega \quad (57)$$

Finally:

$R_{circ.suppl.}$  – supply system resistance: this is equal to sum of feeder cable ( $R_{fc}$ ) and wire ( $R_l$ ) resistances, rails’ resistance ( $R_{rail}$ ) and return cables resistance ( $R_{rc}$ )

$$R_{\text{circ. suppl}} = R_c + R_l + R_{\text{rail}} + R_{l\_BP} = 0,0141 + 0,2145 + 0,0139 + 0,0241 = 0,267 \Omega \quad (58)$$

Taking into consideration **present-day** resistance of the contact system, line voltage has been calculated:

$$U_L = 717 - 2000 * 0,0255 - 1400 * 0,267 = 292,2 \text{ V} \quad (59)$$

Such a low line voltage may come into effect only if the maximum current surges of the feeder in question occur almost simultaneously with high current loads of remaining feeders (of same substation). Since November 2001 116Nd (Citadis 100) trams run on schedule through “Pętla Brynów”. These are trams of modern design, where currents in last stage of start-up as well as auxiliaries’ rated loads may be as high as 1100A. Hence the probability of superimposing current surges of investigated feeder caused by two or more trams with high currents of remaining feeders. In consequence, line voltage drops below 400V, which in turn hinders the operation of 116Nd trams (their minimum supply voltage is 400V).

Current surges shown in Fig. 12 and exceeding 1000A are usually the sum of load currents of 116Nd and 2x105Na trams. The negative values of feeder current correspond to 116Nd tram recuperative braking current, flowing through substation d.c. bus-bars into other sections of contact system supplied from “Park Kościuszki” substation.

The balance of energy consumption, which has been calculated on the basis of measured current load courses of “Park Kościuszki” feeders, has shown that braking energy flowing from a section, where there is no momentary demand for excess energy, into other sections supplied from traction substation, constitutes *c.* 2,4% of total energy supplied by the substation to the contact system. For “Pętla Brynów” contact system section this energy is as low as 0,3% of total energy.

Increasing this value, which is equivalent to increasing recuperated energy utilisation by other trams, running in other contact system sections, would be possible if following conditions were met:

- a) supply system were modernised (in particular tram’s supply circuits resistance reduced, new feeder cables laid, contact system section divided into shorter sub-sections ),
- b) new substations were constructed (adopted to feeding excess braking recuperated energy back into 15-kV power network),
- c) energy accumulator backed supply system.

Setting-up new traction substations with the goal of reducing their supply areas will not increase recuperated energy utilisation, since at present<sup>9</sup> there is no possibility of feeding excess energy back into power network. Neither it is possible to supply trams running in neighbouring substations’ supply areas with this energy. However, if supply sections were shorter, then a smaller number of trams would run simultaneously in a given section. Hence the probability of overlaying start-ups and braking of trams running in the same section is increased and, as a result, more starting trams would be able to use recuperated braking energy.

### Supply system diagrams – present-day condition, modernisation schemes

Possible approaches to supply system modification are as follows:

- standard approach, that is decreasing supply system resistance (by increasing cross-sections of catenary wires and decreasing distances between substations),

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<sup>9</sup> Recuperating energy into power network is hindered, since energy does not fulfil necessary technical requirements and there are also legal problems related to signing contracts obliging the energy provider to recuperate braking energy

- equipping traction substations with power electronics converters, making possible recovery of energy into the power network<sup>10</sup>, (provided that energy company /GZE S.A./agrees),
- adopting new energy source – an energy accumulator, improving braking energy recuperation efficiency (energy is consumed by starting trams) and supply conditions as well<sup>11</sup>. This method has not been utilised in Poland so far.

In case of modern trams with recuperative braking, running on steep grades, it is not possible to calculate energy consumption by analytical equations only. The reasons can be stated as follows:

- 1) recuperated energy may be fed into braking tram's own auxiliaries and, within certain limits, it may be also fed to other trams running in the supply area of traction substation in question,
- 2) factors limiting recuperation are:
  - a) resistance of contact system section between braking tram and tram being supplied from contact system at the same time instant,
  - b) traction substation d.c. bus-bar voltage,
  - c) contact system allowable maximum voltage.
- 3) factor influencing braking energy recuperation is demand for electric energy created by other (adjacent) trams running at a given time instant (this is true in particular since there is no possibility of feeding recuperated energy back into the a.c. 15 kV power network)<sup>12</sup>.

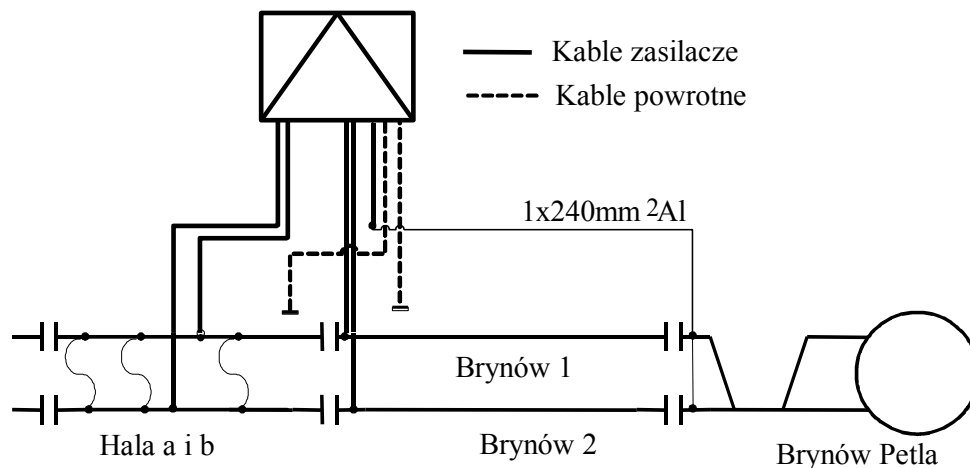


Fig. 13 Present-day supply system scheme. Cables not identified in the picture are YAKY 625mm<sup>2</sup> (kable zasilacze – feeder cables, kable powrotne – return cables)

Possible modernisation schemes for contact system supply are given in this chapter. At every stage of standard (conventional) modernisation (laying out of new cables<sup>13</sup>), it has been

<sup>10</sup> cf. MIERZEJEWSKI L., SZELĄG A.: *Wpływ taboru z rekuperacją energii na warunki funkcjonowania systemu elektroenergetyki trakcyjnej prądu stałego*, V Międzynarodowa Konferencja Naukowa Trakcja Elektryczna w Transporcie Regionalnym, MET'2001, Gdańsk 31 May-2 June 2001r. (*Impact of rolling stock with energy recuperation on operating conditions of d.c. traction power plants*. In Polish), s.11-1.

<sup>11</sup> cf. KAŁUŻA A.: *Techniczno-ekonomiczna analiza zastosowania zasobnika w celu zwiększenia efektywności użytkowania energii w infrastrukturze transportu miejskiego*, TTS nr 5/6 2002 r. (*Using energy accumulator to increase energy effectiveness in city transport – technical and economical feasibility analysis*. In Polish).

<sup>12</sup> Energy recuperated during braking cannot be re-sold to the energy provider (in this particular case GZE S.A). This energy does not fulfil quality requirements and this hinders signing of contracts. There are no devices in the traction substations making possible the return flow of energy into the power network.

supplemented by proposed introduction of energy accumulator. Modernisation is indispensable to proper modern rolling stock operation.

Different modifications of supply system have been investigated as follows:

- 1) Supply system as of today (Fig. 13).
- 2) Simple modernisation of supply system – feeder cable to Brynów Pętla of **smaller resistance** is used (Fig. 14- variant II),
- 3) Possible modernisation variant III. Supply system scheme. Complex modernisation of supply system.

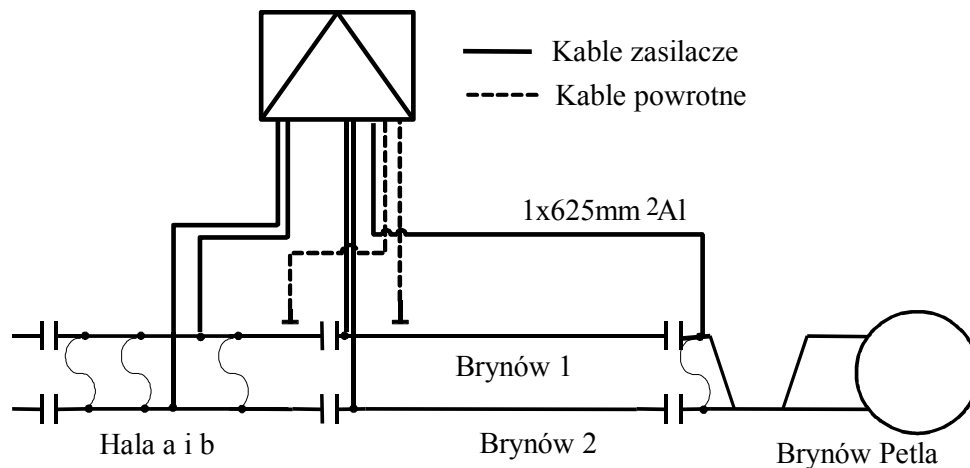


Fig. 14 Modernisation variant II. Present-day supply system scheme and new feeder cable of “Pętla Brynów” section. Cables not identified in the picture are YAKY 625mm<sup>2</sup>- simulation

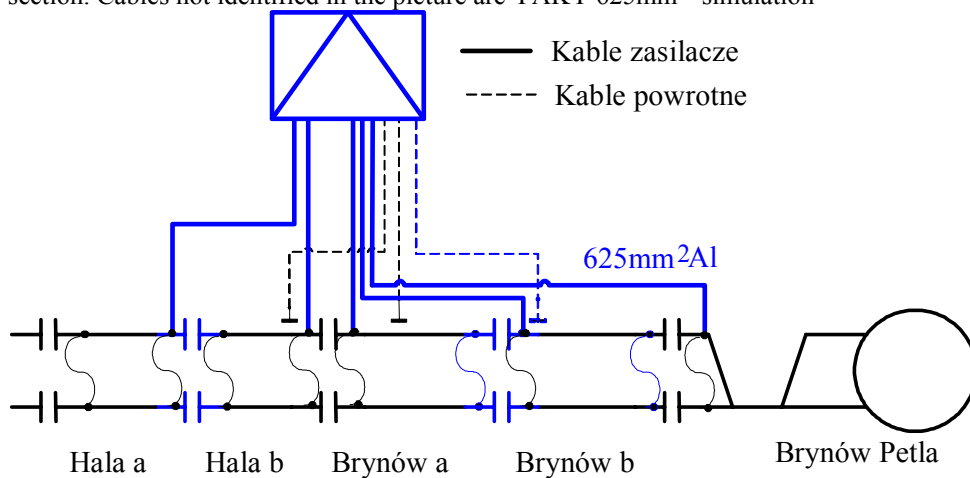


Fig. 15 Modernisation variant III. Supply system scheme. Cables outgoing from substation are denoted on the left by symbols fc1, fc2, fc3, fc4, fc5, rc1, rc2, rc3 (625mm<sup>2</sup> Al cables)

Fig. 15 depicts a standard, conventional tram supply system design. The following alterations have been introduced:

- I) Two section isolators have been added, dividing section “Hala a & b” and setting up two new separate sections: „Hala a” and „Hala b”. Isolators dividing section “Hala a & b” into „Hala a” and „Hala b” should be located in the vicinity of a point, where tracks

<sup>13</sup> reduction in resistance of contact line and feeder cables in braking current path – between braking tram and tram being supplied by recuperated energy – is necessary to improve supply conditions and facilitate energy transmission, i.e. extending tram recuperated braking energy utilisation.

intersect Poniatowski Street. Connection of one of feeder cables to section “Hala b” has been modified in order to improve short-circuit protection.

- II) Contact lines over left and right track have been joined, transforming two long sections „Brynów 1” and „Brynów 2” into one section „Brynów”. This section in turn has been sub-divided into two short parts – called “Brynów a” and “Brynów b”.

Laying out of new cables in supply system and shortening of contact system sections will reduce tram supply circuits resistance. Reduced resistance brings about following benefits:

- 1) voltage drops are decreased (and, by the same token, power losses in contact system go down)
- 2) and since it is indispensable to:
  - a) provide the end terminals of contact system with voltage not less than minimum continuous voltage, i.e. 400V,
  - b) not exceed maximum allowable line voltage ( $U_{LBmax} = 720V$  for recuperative braking)

voltage drops are decreased, hence it is possible to limit necessary traction substation d.c. bus-bar voltage  $U_{sub}$  from *c.* 700 to 660 V.

Reduction in d.c. bus-bar voltage in traction substation increases possible amount of recuperated braking energy, which can be transmitted over contact line and used by other trams.

Modernisation of the supply system will enable increase in amount of energy recuperated, due to reduction of supply circuit resistance and increase in voltage difference between substation and braking tram. These interrelations are specified in the model (Fig. 4) and Eq. (12), where maximum tram braking current is equal to  $IB = [ULB - U_{sub} + IJ * (Rc + Rs1)] / (Rc + Rs1 + Rs2)$ . Braking current IB value depends on  $IJ^{14}$ , i.e. on the current supplied to another tram, on total resistance of feeder and return cable (YAKY625mm2 resistance is  $Rc$ , contact line resistance and resistance of rails between braking tram and tram using recuperated energy is  $Rs2$  (cf. Fig. 3, Fig. 4)). Therefore reducing  $Rs2$  as well as increasing difference ( $ULB - U_{sub}$ ) raises the value of recuperated braking current IB, which may then be utilised anew (also braking energy utilisation).

**Example No. 1b. (cf. Example No.1. Calculation of maximum braking current of tram No.604; tram was leaving contact system section “Brynów 2” (Fig. 8; 5:02:13 hrs) )**

In order to show how the conditions of transmitting recuperated braking energy over modernised contact system improve (in accordance with Fig. 2), maximum braking currents of trams presented in previous examples have been re-calculated.

Calculation of braking current of No.604 tram, leaving “Brynów b” section of contact system (braking occurs when tram accesses “Brynów Słowików” stop – cf. Fig. 8, 5:02:13 hrs). Substation load parameters are:  $I_p = I_{sub} = 621A$ ,  $U_{sub} = 701V$ .

In example 1, calculated value of braking current, given existing supply scheme configuration has been equal to:

$$IB = IB_{max} = [ULB_{max} - U_{sub} + I_j(Rc + Rs1)] / (Rc + Rs1 + Rs2) = (720 - 701 + 0) / 0,2211 = 85,9 \_A \quad (60)$$

To calculate improvement in transmitting and utilising braking energy, a different (increased) value of current, which can flow in modernised contact system (cf. Fig. 15) under comparable

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<sup>14</sup>  $U_{sB}$  – tram braking voltage at a point, where current  $I_B$  from the braking tram is fed back into contact system



load conditions are determined. Symbols used here are the same as in Fig. 3, Fig. 4, it has been assumed that  $I_j=0$  and  $R_{s1}=0$  (tram No.1 is absent).

Maximum braking current  $IB_{max}''$ , calculated in accordance with (12), assuming that  $ULB_{max}=720V$ ;  $U_{sub}=701V$ ;  $I_{sub}=621A$ ;  $I_j=0$ ;  $R_{s1}=0$ , in modernised supply system will be equal to:

$$IB_{max}'' = [ULB_{max} - U_{sub}] / (R_c'' + R_{s2}'') = (720 - 701) / (0,12655) = 150A \quad (61)$$

Additionally, taking into account the possibility of voltage decrease from 701V to 661V at traction substation d.c. bus-bars, it can be shown that:

$$IB_{max}'' = (720 - 661) / 0,12655 = 466A \quad (<621A) \quad (62)$$

Calculation results show unequivocally that recuperative braking current increases (from 85 A to 466 A), hence braking power and energy which can be recuperated due to modernisation of traction supply system also increase.

This current is less than current supplied from substation to other trams (621A). Therefore, there is no necessity to buy extra energy.

### **Energy accumulator used to improve energy consumption effectiveness in city transport – feasibility analysis**

Putting modern trams into operation creates the need for modernising supply system. Either the supply system may be developed (additional traction substations, feeder and return cables put in) or autonomous supply sources introduced (i.e. independent of local energy provider - e.g. energy accumulator or other energy source). It will ensure reduction in voltage drops and, at the same time, improve efficiency of recuperative braking. Usually additional substations are set up and new feeder cables, with increased cross-sections are laid. Efficiency of recuperative braking can be improved by changes in contact system sectioning, increasing feeders' cross-sections and decreasing traction substation bus-bar voltage.

Since financial investments (related to modernisation of supply system) are significant, it might be interesting to consider other ways of backing up tram contact system.

One possible method concerns introduction of energy accumulators instead of additional traction substations or elongated new feeder and return cables.

Essential gain in recuperative braking efficiency can be attained by introducing additional accumulative energy sources into trams' supply system<sup>15</sup>.

Taking into account introduction of energy accumulator into tram supply system<sup>16</sup>, it is possible to reduce both voltage drops in supply system and load current surges as well. This technical solution may also improve the effectiveness of recuperative braking, which is equivalent to energy consumption decrease in trams running within the area, where energy accumulator operates.

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<sup>15</sup> cf. KAŁUŻA A., SIKORA A.: Założenia i wyniki symulacji przebiegów ruchu oraz przebiegów parametrów elektromechanicznych pojazdu napędzanego silnikami indukcyjnymi, Zeszyty Naukowe Politechniki Śląskiej nr 1471, Zeszyt ELEKTRYKA z.173, Gliwice 2000 r., s.17-27. (Assumptions and run simulation results – operation and electromechanical parameters of a vehicle driven by induction motors. In Polish) .

<sup>16</sup> cf. KAŁUŻA A.: Successful simplified model for energy-efficiency improvement analysis of tramways operating in selected section of traction substation supply area. Impact of energy accumulator on supply conditions and trams energy consumption, XX. Sesit Katedry Teoretickej Elektrotechniky, VSB - Technická Univerzita Ostrava, Ostrava 2002

Since standard modernisation of supply system has been planned for a long time (a new site for a substation has been selected long ago), additional financial outlay necessary to energy accumulator purchase could not cover the costs of standard modernisation.

This type of modernisation could be in particular adopted in these contact system sections, which are characterised by:

- long distance from the substation, which implies increased resistance of the supply system
- location of many tram stops and other points along the route requiring tram's running to a standstill (energy can be recuperated during necessary braking)
- existence of many downhill slopes of significant gradients, which demand braking in order to keep tram's speed constant.

Quite a lot of contact system supply sections of GOP<sup>17</sup> tram lines correspond closely to characteristics given above.

## Conclusion

Presented analysis results in determination of benefits arising from modernising one contact system section supply.

If whole supply system were modernised, the amount of recuperated braking energy would greatly increase (it is shown by analysis of a longer route section, containing steep slope).

This is useful to search not only for the ways to limit the energy consumption, but also for the means to compensate the supply system loads, in order to limit the contracted power.

However, simulating a complex system like that would require additional assumptions, such as possibility of transferring energy between remaining feeders in contact system section supplied from "Park Kościuszki" substation, statistical analysis of different route sections loads, simultaneous incidence of trams braking and starting and rather more developed equivalent schemes.

A whole new field of theoretical research opens here. At the same time, significant financial outlay would be indispensable for test measurements, analysis and modernisation of the whole supply system.

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<sup>17</sup> GOP-Górnos Śląski Okręg Przemysłowy - Upper Silesia Industrial Region

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