Stator Deflection Shapes of Electrical Motors as a Source of Acoustic Noise

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Abstract – **The natural mode shapes associated with structural resonances can differ significantly from vibration responses of a stator structure excited by a series of rotating radial magnetic force waves. The modal analysis has been used in order to obtain stator mode shapes. For acquiring operating deflection shapes of the stator structure under steady state operation of the motor, the operational deflection shapes analysis has been applied. Complex operating deflection shapes of the stator, longitudinal and circumferential, have been compared with mode shapes and the results are presented in this paper. The importance of the decomposition of stator circumferential operating deflection shapes for the purpose of acoustical calculations has been indicated. Basing on vibration measurements of a low-power induction motor, the authors show that not only natural mode shapes, but also operating deflection shapes, can have significant effect on the sound power level of the acoustic emissions of induction motors. Difficulties and recommendations concerning the measurement methodology of operating deflection shapes are also discussed in this paper.**

1. Introduction

The main source of acoustic noise from rail transport is usually the wheel-to-track interactions [1]. But this is not true of vehicles at a stop, or starting, or running at low speed; in those cases, the predominant noise contribution is by the power/drive unit. The electricity-operated rail vehicles, especially modern constructions, are often equipped with AC motor drives fed from PWM inverters. Virtually all components of such drives can have some direct or indirect effect on the overall acoustic noise emission, as illustrated in Fig.1, but the main share will be attributable to the driving motor. In this connection, it is important to understand the vibro-acoustic properties of the AC motors, and to develop modelling and measurement methods with which to design and assess the motors and drives. This paper offers some insight into certain generic vibro-acoustic properties of induction motors, as well as their measurement and analysis.

2. Electromagnetic Noise of Induction Motor

From a reason of specific constructional features, and mostly because of a relatively short air-gap, that prevailing source of acoustic noise emitted to the environment by induction motors are magnetic phenomena related to higher field harmonics in air-gap. In case of squirrel-cage induction motors these phenomena are usually intensified at power supply with distorted

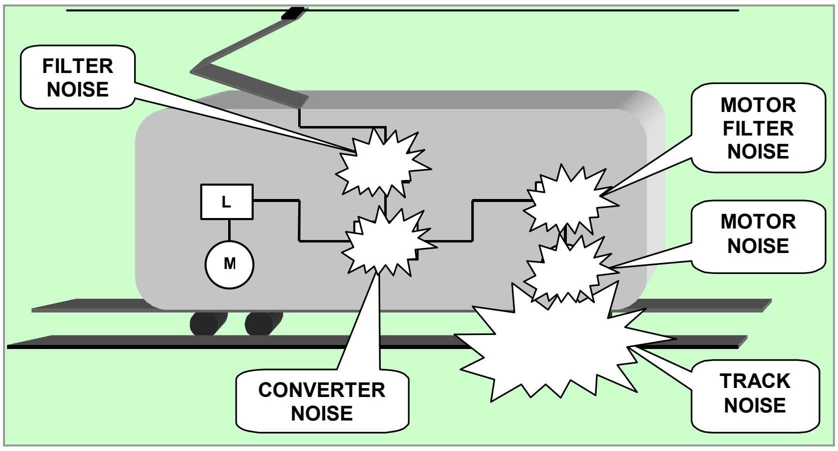


Fig.1. Main causes of acoustic noise on electric rail vehicle

voltage. The main reason of noise of magnetic origin are periodical deformations i.e. vibration of the motor external surface, as the result of varying in time radius stresses operating on internal surface of the hole of stator core. Assuming that magnetic permeability of the machine core μ_{Fe}=:, instantaneous values of circumferential distribution of radius stresses operating on the internal stator surface of induction motor can be defined with the following formula:

$$\sigma_{ra}(\varphi,t) = \frac{b^2(\varphi,t)}{2 \cdot \mu_0} \tag{1}$$

where: $B(\varphi,t)$ - instantaneous values of circumferential distribution of flux density in the air-gap,
φ - angular position on the circumference of the internal stator surface,
μ_o - magnetic permeability of free space,
t - time.

Mechanical stresses on the internal surface of stator core are therefore a function of square of the flux density. The flux-density in the air-gap depends on values and distribution of flux in the motor and on the local magnetic permeability, according to general equation:

$$B(\varphi,t) = \Theta(\varphi,t) \cdot \Lambda(\varphi,t) \tag{2}$$

where: $\Theta(\varphi,t)$ - m.m.f. distribution dependent, among others, on time - distribution of current,
$\Lambda(\varphi,t)$ - magnetic permeance.

Therefore values and spatial distribution of radius stresses described with equation (1) depend on number of phases, number and configuration of slots, distribution of coils in winding, properties of iron and harmonic contents of feeding current.
The acoustical calculation (e.g. of sound power level) for induction motors involves understanding of interaction between magnetic forces (1) and the stator mechanical behaviour. In practice these calculations could be made likewise Zhu and Howe did in [2] – using the formula (3):

$$W_a = \pi \cdot c \cdot \rho \cdot V_r^2 \cdot D \cdot \frac{L}{2} \cdot I_r\left(n, m, f, \frac{L}{D}\right) \tag{3}$$

where: c, ρ - sound speed in air and density of air;

V_r - r.m.s. velocity of radial vibrations of the stator's frame outer surface;

D, L - outer diameter and length of the stator frame;

$I_r(n, m, f, L/D)$ - relative sound intensity coefficient as a complex function of: circumferential radial vibration shape – n; longitudinal vibration shape – m; frequency of vibration – f and length to diameter ratio.

In the past computations of acoustical behaviour of electrical machines have concentrated on identifying stators' natural frequencies and mode shapes and on modelling conditions of acoustical waves emission. Over the years numerous researchers have attempted to calculate the sound power level from the relative sound intensity coefficient with respect to the motor dimensions, natural mode shapes (with their frequencies) and the stator vibrational radial displacement. Natural mode shapes are associated with structural resonances of the motor's stator. Resonant vibration is caused by the interaction between the inertial and elastic properties of the materials within the stator structure. For the majority of low-power induction motors produced for general purpose use stator resonant frequencies with significant amplitudes are in the frequency range from about 500Hz to 5kHz, and the "pure" circumferential mode numbers are equal 0, 1, 2, 3 and 4.
On the contrary, series of rotating radial magnetic force waves caused, for example, by slot harmonics, saturation harmonics, eccentricity harmonics, current harmonics have (theoretically) various mode numbers at different frequencies from the acoustical range. These forces excite stator structure vibration responses which have been named operating deflection shapes (ODS). ODS can differ significantly from natural mode shapes of the stator. They depend on acting forces and material properties as a result of failure to maintain tolerances, material failure, premature fatigue [3] and, what is very important, they can also be defined for structures that don't resonate.
The main objective of the paper is to show, how to make decomposition of obtained ODSs in order to acquire their component mode shapes. This way, calculation of electrical drives' sound power levels can be performed as like for a single mode shape.

3. Investigation of stator deflection shapes

3.1. Investigation Method

To determine and animate deflection shapes for discrete frequencies of a vibration spectrum of the structure the modal analysis (MA), as one of the methods, can be used.
To visualise (i.e. animate changes in geometry) structure vibrations that exist in a machine at work, and are the result of real working conditions, ODSs are being determined. ODSs can be calculated from the formula(4):

$$\{ODS(j\omega_0)\} = [H(j\omega_0)]\{F(j\omega_0)\} \tag{4}$$

which describes the structure response to the exciting force F for a given frequency ($j\omega_0$), where H is a matrix of frequency response function (FRF).
The formula (4) stands for classical [4] ODS definition in frequency domain. Variable in time deflections obtained in that way describe current periodic movements of the structure (ODS)

for a given frequency. From ODS only relative amplitudes and phases between signals from different measurement points can be determined.

Measurements of ODSs have been carried out with the help of one reference transducer and a number of fixed mounted or movable measuring transducers. Amplitudes and phases for each measuring point in comparison to the reference point are being determined in that way. ODSs can be determined either from many structure responses in time domain measured simultaneously or from one out of many different measurements in frequency domain [3]: FRF, transmittance and modern operational modal analysis (OMA).

The latest investigation method is the analysis that is based only on responses measurements. External exciting forces acting on a machine at work in normal working conditions are treated as input signals that are not measured. This method in the literature references have been named ODS FRF – e.g. by Richardson [4] in 1997. ODS FRF can be performed by means of operations carried out on vibration signal spectra. A power cross spectrum contains a relative phase between two responses whereas an auto spectrum contains precise amplitude of one of the responses. When two signal responses are measured simultaneously the phase between them can be determined. Using a dual-channel analyser power cross spectrum between each signal response and a reference signal response are being determined. Scaled ODS FRF points (absolute values) are created by the amplitude change of each cross spectrum with auto spectrum of the response. The operation can be written as:

$$\{ODSFRF(j\omega)\} = \frac{G_{o,x}(j\omega) \cdot G_{x,x}(j\omega)}{\left|G_{o,x}(j\omega)\right|} \quad (5)$$

where: $G_{o,x}(j\omega)$- average power cross spectrum between the reference signal (o) and the response signal in the point x;

$G_{x,x}(j\omega)$- average power auto spectrum of the response in the point x; $j\omega$ - frequency variable.

3.2. Laboratory Investigation Results

Within the scope of ODS FRF, measurements of vibrations auto and cross spectra for all measurement points in relation to one reference point for ODSs have been performed during no-load running of the motor.

In order to prepare two 5.5kW induction motors for investigation measurements some ventilation ribs have been removed and these new surfaces have been prepared for mounting piezoelectric accelerometers – 90 points in total, distributed as shown in Fig.2. During the investigation the motors were hanged by feet using springs. Before the investigation the motors were worked ~~heated~~ until they reached the fixed working temperature of the machine's frame and bearings.

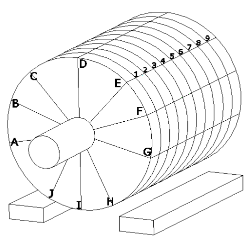


Fig.2. Location of 90 vibration measurement points on the motor's outer stator surface

An example of measured spectrum is presented in Fig.3. As results from Fig.3, for sinusoidal supply, forces of electromagnetic origin generate radial vibrations of the stator outer surface with the following dominant frequency components: 24, 100, 538, 800, 900, 1232, 1600, 2172, 2256 and 2392Hz. From the comparison of these frequency values with FRF spectra of the stator vibration, obtained earlier and reported in [5], it can be concluded that only some of them could be caused by resonant excitation of the stator structure.

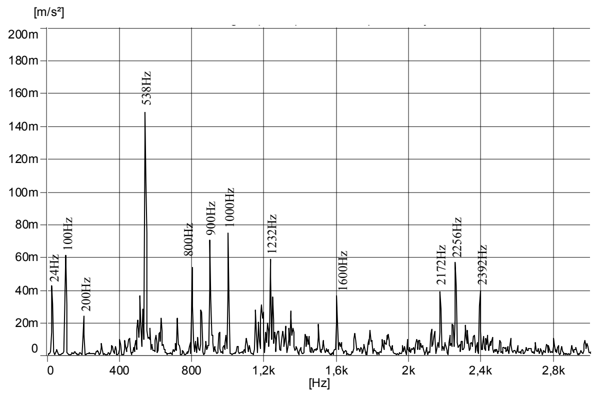
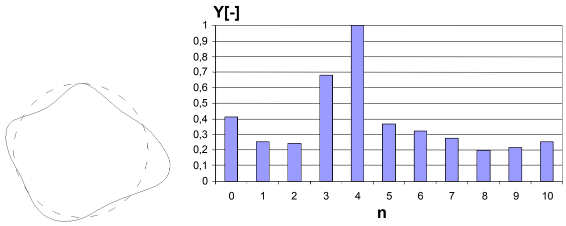


Fig.3. Example of radial acceleration spectrum for the stator outer surface of 5.5kW motor (2p=4, no-load, mains supply)

Despite the fact that the results have been obtained for 90 points it was necessary to apply a procedure using curve-fitting algorithm. Using this procedure distribution of deflections as a function of two dimensions – along the circumference and along the generatrix of the motor's frame – have been obtained. In order to determine components of complex deflection shapes of the frame a decomposition procedure using discrete Fourier transform (DFT) in the frame circumference domain has been performed.

An example of decomposition has been made for the deflection shape that has arisen as a result of excitation of the stator structure by a significant force of a frequency far from its resonant frequency. In Fig.4 circumferential ODS of the frame, with the frequency of 100Hz, caused by the stress being a result of the basic field harmonic operation is presented. This shape for the motor under investigation should have the mode number n=2p=4. Meanwhile, as can be seen from Fig.4, it is also a resultant of shapes with other mode numbers. The decomposition of the ODS presented in Fig.4 shows that, apart from the basic mode number n=4, other shapes with mode numbers 3, 0, 5 … are also present.



a) b)

Fig.4. Circumferential ODS for the frequency of 100Hz in the centre of the frame length (measurement points A5, B5, ... J5) – a) & result of decomposition - b)

Chosen results of ODS FRF for frequencies equal 538Hz, 900Hz and 1232Hz are compared in Fig.5 with the results of MA obtained for the same frequencies and reported in [5]. In consecutive columns (from the left hand side) circumferential deflection shapes for all nine measuring surfaces are shown. In the last column deflections along the frame generatrix are shown. Comparison of these results are presents also in table 1.

As follows from Fig.5 circumferential modal deflection shapes are almost regular with determined mode numbers (node pairs). Changes in deflection shapes along the frame generatrix are also small. On the other hand circumferential ODSs can be characterised as complex shapes for which determination of the mode number is difficult, if generally possible. The resonance frequency of 538Hz is an exception. For this frequency the circumferential mode shape with the mode number n=1, during operation of the motor is only slightly modulated. The longitudinal modal deflection shape with the mode number m=0, during operation of the motor is retained.

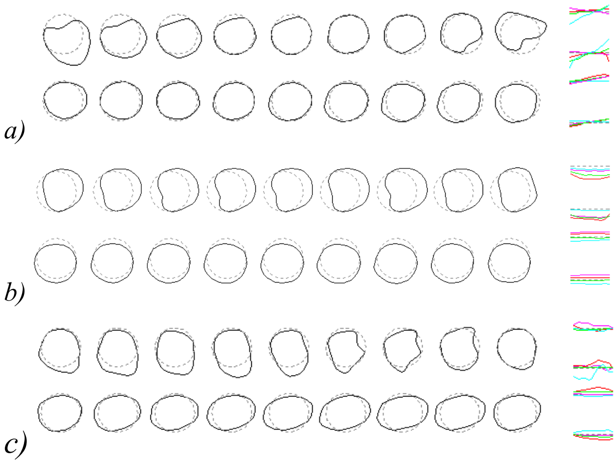


Fig.5. Selected operating and modal deflection shapes of the stator for a) 538Hz, b) 900Hz, c) 1232Hz

The circumferential ODS for 900Hz in comparison to the modal shape is enhanced near the bearing shields – points 1, 2 and 8, 9; at the same time both deflection shapes – longitudinal modal and operating – have the mode number m=1. The comparison between decomposition of deflection shapes near the bearing shields for the frequency of 900Hz (Fig.6) shows that the shape with the dominant mode numbers n=1, n=2, during operation of the machine, changes into the shape with n=1 with a smaller component with n=0.

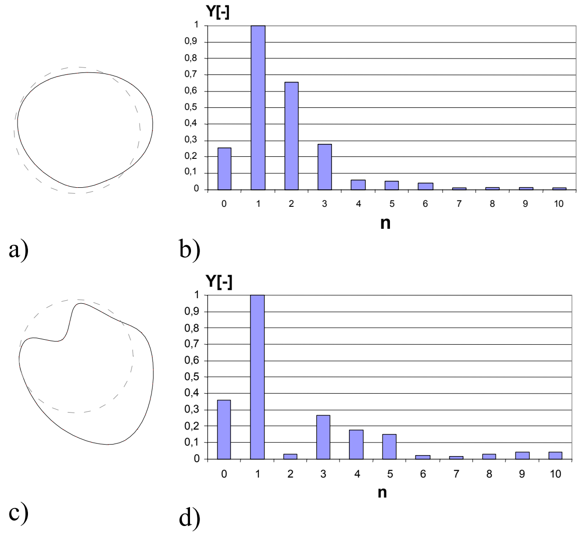


Fig.6. Modal circumferential deflection shape – a) & result of decomposition - b); ODS – c) & result of decomposition – d) for the frequency of 900Hz in the centre of the frame length (measurement points A1, B1, ... J1)

Near the frequency of 1232Hz the circumferential mode shape with the mode number n=2 changes into the ODS of a similar shape. The biggest changes in shape, in contrast to the frequency of 900Hz, can be observed in the middle part of the frame – surfaces 2, 3 and 4.

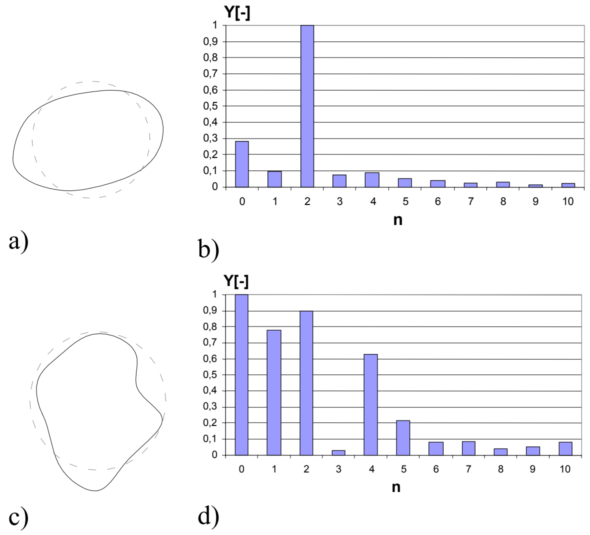


Fig.7 Modal circumferential deflection shape – a) & result of decomposition - b); ODS – c) & result of decomposition – d) for the frequency of 1232Hz in the centre of the frame length (measurement points A5, B5, ... J5)

The longitudinal deflection shapes' mode number m=0, for operation of the motor, changes into higher mode numbers m=1 or m=2. Analysis of the decomposition results presented in Fig.7 shows how the stator structure can be a subject to deformation for the excitation from the resonance frequency range. For the resonance frequency of 1232Hz the modal circumferential deflection shape contains one "pure" dominant wave with the mode number n=2. Meanwhile for the machine at work the ODS contains waves with mode numbers n=0, 2, 1 and 4.

Table 1: *Operating deflection shapes of no-load, sinusoidal supplied induction motor's frame and modal deflection shapes for predominant components of radial vibration – experimental results*

	Operational Deflection Shapes		Modal Shapes	
Freq. **f** [Hz]	Circumferential Mode **n**	Longitudinal Mode **m**	Circumferential Mode **n**	Longitudinal Mode **m**
24	1	1	-	-
100	4; 3(68%); 0(41%)	1	-	-
512	1	1	-	-
538	1	0	1	0
628	1; 0(48%), 3(37%)	2; 3	-	-
900	1; 0 (37%)	1	1; 2(65%)	1
1000	4; 0(82%); 2(59%)	2	-	-
1008	2; 0(22%)	-	-	-
1182	1; 2(61%)	0; 2	-	-
1232	0; 2(90%); 1(81%); 4(58%)	1	2; 0 (<30%)	0
1344	0; 1 (53%)	0; 2	-	-
1436	2; 1(95%); 0(62%); 3(58%)	-	-	-
1600	2; 1(96%); 4(73%), 3(65%)	1	-	-

4. Conclusions

As a result of the investigation of a low-power induction motor using ODS FRF analysis and modal analysis (MA) it was proved that dominant components of the frame vibration spectrum do not have to be correlated with the highest deformability of the stator mechanical structure. An electromagnetic force of a considerable value applied to a stator results in dominant stator deflections also outside the range of the stator's resonant frequencies.

In contrast to a few "pure" modal shapes determined from MA (SISO – single-input single-output) circumferential and longitudinal operating deflection shapes (ODSs) are very complex in form.

In order to evaluate complex deflection shapes obtained using ODS FRF their decomposition using discrete Fourier transform (DFT) has been applied. The results of decomposition allowed to conclude that forces with frequencies significantly different from stator's free vibration frequencies, and force circumferential vibration shapes that contain dominant component and additional deflection shapes of higher or lower mode number. On the other hand near the resonant frequencies ODSs have dominant but not "pure" component; moreover the dominant frequency do not have to be associated with the same mode number.

In order to calculate acoustical properties, decomposition of ODSs is necessary. The amplitude of deflections for some frequency values is bigger near the bearing shields. This means that these longitudinal deflection shapes' mode number m equals 2 with one node in the middle of the frame length. For other frequencies the amplitude of deflections is bigger in the middle of the frame length, with a tendency to the mode number m=1. The above observations mean that this have to be taken into consideration in an acoustical model.

Because ODSs can differ from mode shapes, therefore, they can also have significant effects on the sound power level of a motor.

There are also difficulties associated with ODSs measurements of low-power induction motors. To obtain data of higher modes it is necessary to make measurements for many measurement points. Ten measurement points on a stator circumference allow to determine deflection shape components up to n = 4. The machine's surfaces have to be properly prepared for mounting accelerometers in such a big number of points; it should be bear in mind that changes in the machine's frame can influence its mechanical behaviour. Another aspect is that the procedure of obtaining all data requires many series of measurements while mounting and dismounting accelerometers is a time consuming process.

5. References

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