Experimental design method: an effective tool for transport equipment design and optimization

F. Gillon, P. Brochet

L2EP-Ecole Centrale de Lille BP 48, 59651 Villeneuve d'Ascq , France

frederic.gillon@ec-lille.fr or pascal.brochet@ec-lille.fr

***Abstract* – This paper presents two examples where the experimental design method is used as a design tool. Firstly, screening activity is carried out with tests on an Electric Vehicle. Secondly response surface methodology is applied in finite element model of an electric BDC motor. The advantages appear obvious that are quickness and accuracy.**

1. Introduction

In a world in perpetual innovation, where new products and new equipments appear every year, it is indispensable to have a reliable and fast design procedure [1,2]

In order to improve an existing equipment, it is often necessary to achieve a large amount of tests in order to know and understand how to modify the factors of the studied system. Tests are often expensive in term of time and cost. That is why, it is essential to have a more rigorous approach. The experimental design method offers this rationalization process [3]. The method allows to manage the right number of experiments to achieve the understanding of the physical phenomena. In fact, a lot of factors, which present frequently interactions between them can be involved, hence the problem becomes quickly very complex.

Nowadays, conception and optimization are lying more and more on numerical models built on computer. Actually, numerical models allow to win a precious time of development, nevertheless more the models are complex, more they require powerful data processing and long computing time. Finally, the user of numerical simulation faces the same problems as the experimenter: the management of a large number of simulations to achieve and to provide as soon as possible a reliable answer to a posed question.

The first part of this article is dedicated to the main advantage of the experimental design method and provides some useful references to the reader who wishes to deepen the method. The following parts present two examples of application. The first one uses the experimental design method in the goal of increasing the electric vehicle autonomy through regulating factors of the speed converter. The method is performed to reduce considerably the number of tests to achieve. The second one uses a numerical model built by finite element. This costly model in time of computation is going to be used in order to create a response surface, which offers a fast and accurate access to the data needed.

2. Experimental Design method

Fischer has first introduced the Experimental Design Method in 1925 but its dissemination in business world is mainly due to Taguchi's works in the seventies [4,5,6]. The purpose of the method allows to increase the productivity of experiments, i.e. :

- To minimize the number of **runs**
- To maximize the results **accuracy**
- To determine **significant factors** and interactions between factors
- To appreciate the factors and responses **robustness**

These goals are also those of a designer of electromagnetic devices running simulations or experimenting hardware. But, when using a F.E. model, the duration of a simulation and the error on results arising from the accuracy of meshes or uncertainty on material properties make numerical simulation a process as complex as an experiment.

Traditionally, an experimenter wanting to know the influence of several factors on a given response varies separately every factor. Doing that, he can't make evident interactions between factors. To show interactions, a solution is to build a grid on the domain of study and to realize one trial at each node of the grid. This solution becomes expansive when the number of factors increases. The experimental design method brings a rigorous method to select a reduced number of points among those of the previous grid. Points are chosen for their statistical properties and several factors are modified at every new trial.

Mainly, the experimental design method allows two actions:

- In **Screening** activity, the designer determines which factors significantly affect the studied response. [7,8]

- In **R.S.M.** activity, precise analytical models are built that can then be easily used in an optimization process [9,10,11].

The experimental design method is also used in quality process and to increase the robustness of industrial devices or production tools [12,13]

3. Experimental Design method used with practical tests

The first example describes the optimization of the factors controlling electronic converter used in a small electric vehicle. The converter drives a Dc Motor with a separate excitation [14].

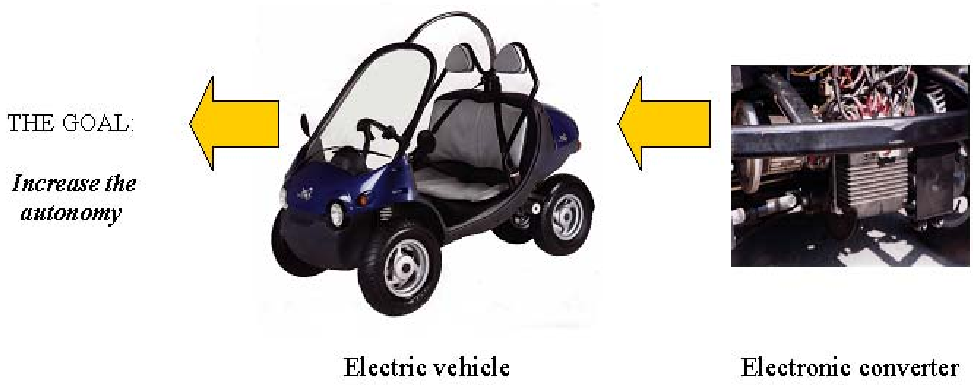


Fig. 1. Small electric vehicle with its electronic converter

3.1 Define the problem

The goal is to increase the electric vehicle autonomy. The electronic converter is a programmable converter, which offers numerous possibilities. Four modes can be warped and for each of these modes, many factors can be adjusted. The most important are shown on table 1. Eight factors of the electronic converter that could have some influence on the current used by the vehicle are chosen. In fact, the energy consumed by batteries is frequently presented in ampere hour, therefore all factors with a supposed effect on the current must be kept.

Table 1: Main electronic converter factors

N°	Name	Description
1	Drive C/L	Maximum current in the motor armature(A)
2	Accel rate	Rate of acceleration (s)
3	Quick start	Starting help (with tension) (/)
4	Max speed	Limit speed (maximal tension) (%)
5	Regen speed	Level of speed to have recuperative brake (%)
6	Field map start	(A) adjustment of flux in motors (%)
7	Field Map	
8	Current ratio	Starting help (overtaking current) (/)

But this converter is like a black box and it's very difficult to know precisely the role of each of these factors or assign them their best values. Tests seem to be the only way. But how to be sure to find the best combination of factors and how long it will take to obtain such a result [15]? The Design of Experiment has been suited to address this kind of problem. Its main purpose is to decrease the number of trails and to allow to choose the good level for each factors and then obtained an optimal adjustment. The Experimental design method helps to the know of the studied phenomenon.

3.2 Building Experimental design

Table 1 factors are numbered to be able to track them down more easily. Each factor can take two values or levels: the highest value quoted "+" and the lowest value quoted "-". As 8 factors are to be tested, if all the tests are performed, it would be necessary to realize 2^8 experiments, that are 256. This is known as a full 2-level factorial design; of course, it's too many tests. Fortunately, a fractional design, available in the theory of Experimental Design requires only 16 tests. With this kind of experimental design the experimenter can know the best level of each factor, nothing less, nothing more. Moreover, experiments used in the experimental design are chosen for their algebraic and statistic properties (orthogonality).

The experiments needed by this design are described in the table 2 where the values assigned to each factor are given for each experiment. This table is the experiments matrix. The fractional design is built starting from the four first factors (1,2,3,4). The experiments matrix is chosen to have an homogeneous distribution of effects confusing [7] : 5 = 234 ; 6 = 134 ; 7 = 123 ; 8 = 124. In fact, interactions between three factors are supposed to be negligible. When the choosen experiments are done, the response can be measured for each line of the matrix (16 lines). It's noticed that shape of the experiments matrix is independent of theresponses.

3.3 Analyze and interpretation

The studied response is the current supplied to the motors which is measured by an electronic acquisition system during a course which is composed of three parts : starting up, maximal acceleration, and then braking and the average current is considered as a standard current consumption. According to effects and interactions calculated in the table 3, significant factors are 1 , 2 , 3, 4 , 5 , 6, 7 , 8 as well as interactions 13 and 14. The effect of the factor *i* on the standard current consumption of the vehicle is called *Ei* and that of the interaction of the factor *i* with the factor *j*, is called *Iij*.

When an effect (or an interaction) is positive, an increase of the corresponding factor (or factors), implies an increase of the current consumption; and inversely if it is negative, a decrease.

A model for the estimated standard current consumption is then set up as:

$$\hat{I} = \text{Mean} + \sum_{i=1}^{8} E_i + I13 + I14 \qquad (1)$$

This model gives a maximal error of 1,44 % (see table 3). Voluntarily, interactions with a too low value aren't taken into account in formula 1.

Table 2: Fractional experimental design 2^{8-4}

Mini	200	0,1	1	70	0	0	0	1
Maxi	350	1	10	100	40	240	50	4
Para	**Courant maxi**	**taux accel**	**quick start**	**vit maxi**	**vit regen**	**fd map start**	**fd map**	**current ratio**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	-	-	-	-	-	-	-	-
2	+	-	-	-	-	+	+	+
3	-	+	-	-	+	-	+	+
4	+	+	-	-	+	+	-	-
5	-	-	+	-	+	+	+	-
6	+	-	+	-	+	-	-	+
7	-	+	+	-	-	+	-	+
8	+	+	+	-	-	-	+	-
9	-	-	-	+	+	+	-	+
10	+	-	-	+	+	-	+	-
11	-	+	-	+	-	+	+	-
12	+	+	-	+	-	-	-	+
13	-	-	+	+	-	-	+	+
14	+	-	+	+	-	+	-	-
15	-	+	+	+	+	-	-	-
16	+	+	+	+	+	+	+	+

Effects	**E1**	**E2**	**E3**	**E4**	**E5**	**E6**	**E7**	**E8**
	+14,71	**-1,08**	**-2,29**	**+3,47**	**-0,25**	**-0,38**	**-0,31**	**-0,04**

Interaction	**I12**	**I13**	**I14**	**I15**	**I16**	**I17**	**I18**
	-0,4	**-1,4**	**+4,1**	**+0,1**	**-0,3**	**-0,2**	**-0,6**

Table 3: Estimate current

Trial N°	**Measured Current (A)**	**Estimated current (A)**	**Trial N°**	**Measured Current (A)**	**Estimated current (A)**
1	96,23	97,61	**9**	95,11	94,94
2	119,29	120,08	**10**	136,76	135,61
3	95,00	94,23	**11**	93,19	92,75
4	118,31	118,13	**12**	133,96	134,49
5	93,40	93,93	**13**	93,95	93,77
6	114,05	113,61	**14**	129,40	128,64
7	93,95	92,81	**15**	91,02	91,82
8	111,59	111,41	**16**	123,88	125,26

To determine which factors are really significant, a variance analysis is required. It allows to calculate from which threshold a factor is significant or not.

As each factor possesses two levels, one degree of freedom (dof) is assigned to each effect and to each interaction. The total number of dof for this design corresponds to the number of tests that's 16. According to the chosen model (1) the number of dof of the model is 11, there are so 5 dof for the residual.

Table 4: ANOVA Test

Effects or interactions	Square sum	ddl	Variance	Fexp	Ftheo 95	Significant
1	3462,82	1	3462,82	1741	6,61	**Yes**
2	18,70	1	18,70	9,4	6,61	**Yes**
3	83,75	1	83,75	42,1	6,61	**Yes**
4	192,28	1	192,28	96,6	6,61	**Yes**
5	1,02	1	1,02	0,5	6,61	No
6	2,27	1	2,27	1,1	6,61	No
7	1,55	1	1,55	0,8	6,61	No
8	0,031	1	0,031	0,01	6,61	No
13	30,8	1	30,8	15,5	6,61	**Yes**
14	272,9	1	272,9	137,3	6,61	**Yes**
Residues	9.9	5	1,98			

The analyze is done by an ANOVA computation carry out in table4. It is dangerous to conclude before to have done this test, because responses are altered by the experimental error then the effects also. Here, four factors and two interactions can be accepted as significant. Naturally, maximal current (factor 1) has the most importance on the consumption of the vehicle. To increase Quick Start (factor 3), the rate of acceleration (factor 2) and to decrease the maximal speed allows to reduce the current consumption during the course.

A similar design of experiences has been performed to study the swiftness of the vehicle and to know which factors are important for a fast starting up. So two modes have been determined: the first one is called fast which is given by this second Design of Experiments while the other one is called economic and has been obtained by the Design of Experiments presented bellow.

In this last case, an autonomy gain of more than 20 km is obtained with only a good factor adjustment of the electronic drive. The design of experiment proves to be an efficient and easy tool to optimize a complex device.

4. Experimental Design method used with simulations

The second example applies the experimental design method to the finite element model of a Brushless Permanent-Magnet Motor in order to increase its output power [16,17,18].

4.1 Building the numerical model

The studied machine is a Brushless Permanent-Magnet Motor with a 24 slots stator. It is a low power motor used in automotive equipment. The armatures geometry can be defined by 9 factors, but a preliminary screening step allows to keep only the 3 main factors. Fig. 2 show the motor structure and the 3 factors, “a” airgap thickness,”Yoke” yoke thickness, “Hslot” slot height. The geometric modifications are applied on a constant domain for which the model can be build. This defines the variation domain for each factor.

With this kind of structure, the E.M.F generated at no load is square-waved so that increasing the E.M.F. wave top value when the phase is supplied (the mean value of the E.M.F. during 120 electric degrees), increases the output power of the machine [19]. The E.M.F is the objective fonction that will be modelled by a response surface to find a maximum or to use as an analytical model to predict a response.

It is obvious that to have reliable results, an accurate numerical model is required. Comparisons between experiment and simulation have proved that this numerical model can be considered as a virtual prototype [1,20].

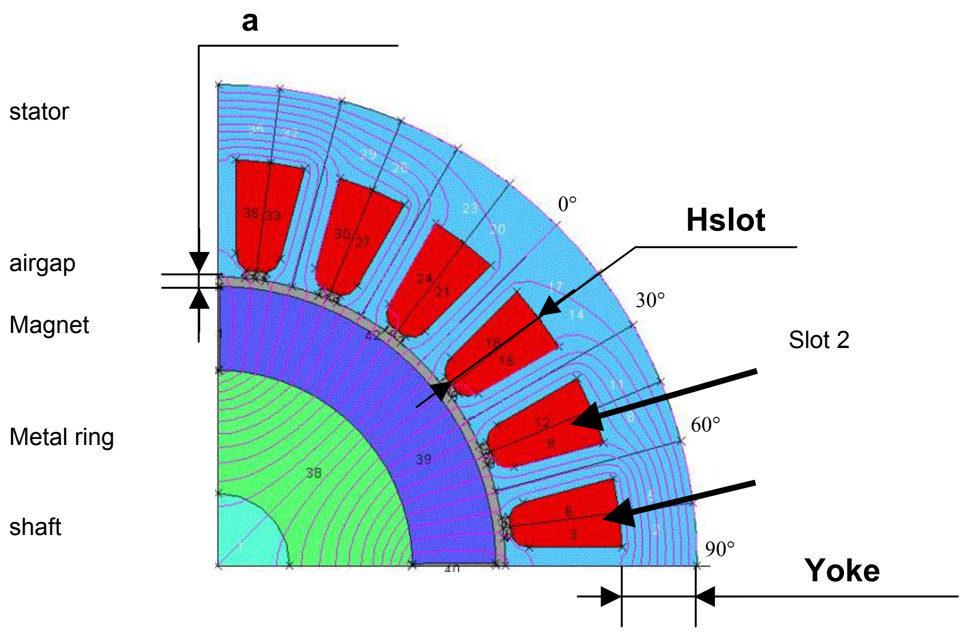


Fig. 2. Structure of the motor and Modelling by Finite Element method

4.2 Response Surface Methodology

The RSM activity purpose is to build accurate analytical models of a complex phenomenon that can depend of numerous factors with or without interactions between them [21,22].

Several available factorial designs are tested in order to appreciate their performances. A full 3 level design, a Box-Behnken, a Central composite and a full 2 level design are compared. All are realised in the same optimisation domain. The second line of the Table 5 illustrates the spatial distribution of simulation points for each design.

Contrary to the screening activity, the RSM design can be applied to a large study domain that will be the validity domain of the constructed model, while the finite element model is reliable.

Full 2 levels factorial design

It is the most simple design and the most economical. The associated analytical model involves all effects and their interactions:

$$\begin{aligned} \hat{Emf} = &\; I + a + Hslot + Yoke && \textit{Mean and Main Effects} \quad (2)\\ &+ a.Hslot + a.Yoke + Hslot.Yoke && \textit{Interaction of order 2}\\ &+ a.hslot.Yoke && \textit{Interaction of order 3} \end{aligned}$$

But it allows to define only a linear polynomial model.

Central composite design

If the full 2 levels factorial design is not enough accurate, some supplementary points can be added to build the central composite design. The building method is then gradual. This new design takes into account the parabolic effects. These supplementary points are included inside the domain boundary of the full 3 level design, so that this set of experimental designs is obtain at low computation cost.

The associated model includes quadratic coefficients (a^2, $Hslot^2$, $Yoke^2$) and neglects third order interaction.

$$\begin{aligned} \hat{Emf} = \; & I + a + Hslot + Yoke && \textit{Mean and Effects} \\ & + a.Hslot + a.Yoke + Hslot.Yoke && \textit{Interactions} \\ & + a^2 + Hslot^2 + Yoke^2 && \textit{Effects}^2 \end{aligned} \quad (3)$$

The coefficients are computed with a classical least squares approximation method.

Box-Behnken

The simulation points of the Box-Behnken design are included in the domain boundary of the full 3 level design but not positioned at the boundary. It is an advantage when certain points are not realisable.

The analytical model associated to the design is the same that the central composite (3) and the coefficients are computed with the same technique .

Full 3 levels factorial design

An alternative solution could be a full factorial design with three levels by factor, but the number of simulations is very important (3^3). But, all these simulations have been also realised in order to compare the three different models.

The analytical model is expressed in term of matrix and vectors. Its computation is simple and fast.

$$\begin{aligned} \hat{Emf} = \; & I + \{a\} + \{Hslot\} + \{Yoke\} && \text{M}\textit{ean and Effects} \\ & + [a.Hslot] + [a.Yoke] + [Hslot.Yoke] && \textit{Interaction of order2} \end{aligned} \quad (4)$$

Fig. 3 shows the response surface of the E.M.F. with *Hslot* fixed at a constant value. The quadratic effect is clearly visible and moreover an inflection point is detectable on the *Yoke* factors.

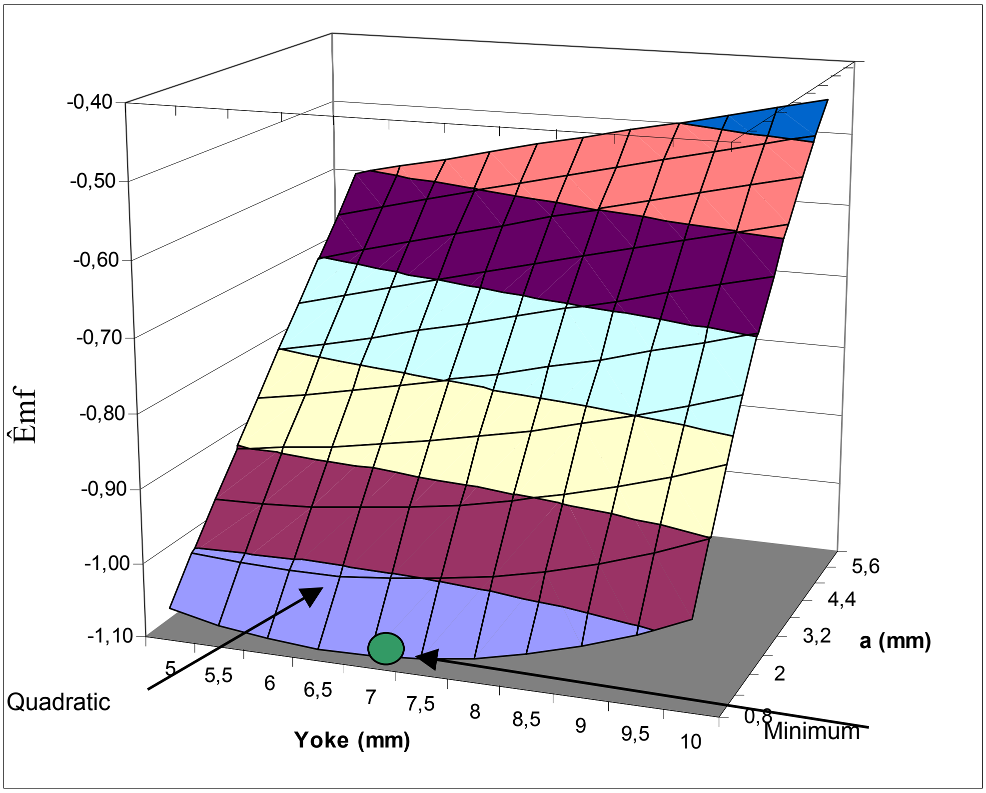


Fig. 3. Response surface of the E.M.F. with h_{slot} fixed at 16.5mm

Comparing the designs

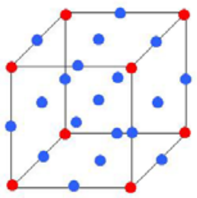
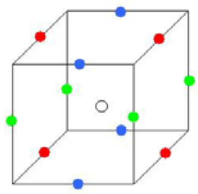
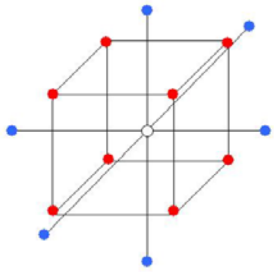
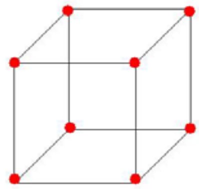
The quality of a design must be appreciated in versus of its cost. The full 3 level design is the most expensive with 27 evaluations or experiments. The Box-Behnken with 13 is very close the central composite design that needs 15 simulations. The cheaper is the 2 level factorial design with only 8 evaluations.

For an important number of factors the use of full 3 levels design become critique, the number of simulation increase very quickly. In this case, the Box-Behnken and central composite design are better adapted.

Table 5 presents the coefficients of the models that have been calculated, except those of the 3 level factorial design, because matrixes and vectors constitute them. The coefficients from Central composite and full 2 level design are very close, in spite of different calculation method. For each experimental design, the maximum error and the R.M.S. error are calculated through the 27 simulation points of the full 3 level design.

The maximum error is important for the 2 level at 59%, but more weak for the central composite design, 40%.In your case, the Box-Behnken design have a good performance with 12% of maximum error. The error for the full three levels design that is the reference design is due to the analytical model where the 3-order interaction is neglected.

Table 5: Model coefficients and model Error

Full three levels	Box-Behnken	Central Composite	Full two levels
 Coefficients			
Mean	-0.7610	-0.7688	-0.7356
A	0.2507	0.1682	0.1750
Yoke	0.0644	0.0361	0.0280
Hslot	0.1811	0.1195	0.1259
a Yoke	0.0150	-0.0051	-0.0051
a hslot	-0.0290	-0.0071	-0.0071
Yoke hslot	0.0137	0.0063	0.0063
a Yoke hslot	/	/	0.0154
a^2	-0.0399	-0.0067	/
$Yoke^2$	0.0196	0.0126	/
$Hslot^2$	-0.0051	0.0114	/
Max Error (%)			
5.76	11.82	37.96	59.44
Rms Error (%)			
2.42	3.37	12.68	17.19

An accurate analytical model is especially interesting for an optimisation process. Using simultaneous valuation on a computer network, and computing in post-processing, several responses, and this method can be very powerful [23,24].

5. Conclusions

Different ways by which the experimental design method can be applied in a design or optimization process have been illustrated. The advantages are obvious, simplicity, quickness, reliability, and accuracy of results. As an efficient tool in industrial or chemical process, it could be also a very useful tool in electrical engineering, particularly in rolling stock or automotive applications.

6. References

[1] S. Brisset, F. Gillon , "The Experimental Design: a tool for Electric Machine Optimization" Emf Marseille, p261-266, Mai 1998

[2] Alberto GARCIA-DIAZ, Don T. PHILLIPS " Principles of experimental design and analysis "Ed. Chapman & Hall – 1995

[3] P. Molinier, B. Belin, P. Brochet, " Compatible CAE standards for equipment design", 6th International Congress EQUIP'AUTO 89 Paris, pp370-376, October 1989

[4] G. E.P. Box, W. Hunter, J. Stuart - Hunter, "Statistics for Experimenters" Wiley Interscience.

[5] Glen Stuart, Peace , "Taguchi Methods", Addison-Wesley Publishing Company, 1992

[6] A. Garcia-Diaz, Don T. Philips, "Principles of experimental design and analysis", Chapman & Hall, 1995.

[7] J. GOUPY "La méthode des plans d'expériences – Optimisation du choix des essais & de l'interprétation des resultants" Ed. Dunod – 1996

[8] M. CALDORA COSTA, S. GIURGEA, J. L. COULOMB, Y. MARECHAL "Application of Experimental Design Method in the Screening of Electromagnetic Devices Parameters" COMPUMAG 2001, Vol. III, pp. 126-127

[9] P. ALOTTO, M. GAGGERO, G. MOLINARI, M. NERVI "Design of Experiment and Statistical Approach to Enhance the Generalized Response Surface Method in the Optimisation of Multiminima Problems" EEE Transactions on magnetics, Vol. 33 N°2, March 1997

[10] D. DYCK, D.A. LOWTHER, Z. MALIK, R. SPENCE, J. NELDER "Response Surface Models of Electromagnetic Devices and their Application to Design" IEEE Transactions on magnetics, Vol. 35 N°3, May 1999

[11] R. RONG, D.A. LOWTHER, Z. MALIK, H. SU, J. NELDER, R. SPENCE "Applying Response Surface Methodology in the Design and Optimization of Electromagnetic Devices" IEEE Transactions on magnetics, Vol. 33 N°2, March 1997

[12] Genichi Taguchi " Introduction to Quality Engineering" Asian Productiotivity Organisation, Tokyo, 1986

[13] Y. K. KIM, J. P. HONG, J. HUR "Application of Response Surface Methodology to Robust Design of BLDC Motor Performance" COMPUMAG 2001, Vol. IV, pp. 148-149

[14] F.Gillon, F. Durand, P. Brochet "Optimization of the performances of a fun electric car", EVS'18 Berlin, 2001, CD

[15] S. Vivier, M. Hecquet and P. Brochet., "Optimisation of linear eddy current brake using designs of numerical experiments" ICEM2000 Finland, pp1785-1789, August 2000

[16] J.P. Yang, Z.J. Lui, S. X. Chen, H.T. Wang and X.K. Gao "Robust Design for cogging torque reduction of Pm Electric Motors using response surface methodology and Taguchi method", pp. 828-829, Compumag Japan, June 2000

[17] X. K. GAO, T. S. LOW, S. X. CHEN, Z. J. LIU "Robust Design for Torque Optimization using Response Surface Methodology" COMPUMAG 2001, Vol. III, pp. 42-43

[18] T. TSUNODA, Y. ISHIHARA, T. TODAKA, K. HIRATA "Geometry optimization of a DC motor by Taguchi method" ISEF 2001, pp. 489-492

[19] T.J.E Miller, "Brushless Permanent-Magnet and Reluctance Motor Drives" Oxford Science Publications.

[20] F.Gillon, P. Brochet, "Optimisation of a brushless permanent magnet motor with the experimental design method", IEEE Trans. On Magnetics, Vol. 34, No 5, pp 3648-3651, Sept. 1998

[21] F.Gillon P. Brochet, "Shape Optimization of a Permanent Magnet Motor using the Experimental Design Method" IEEE Trans. On Magnetics, Vol. 35, No 3, pp 1278-1281, May. 1999

[22] George E.P. BOX Norman R. DRAPER " Empirical model-building and response surfaces" Ed. John Wiley & Sons – 1987

[23] CP Riley, RCF McLatchie, R Janssen, T Gutteriez, J Simkin, P Brochet, C Furmaniak, F.Gillon, G Molinari, P Alotto, JF Lemoine, G Drago, "An Environnement for Electromagnetic Design Optimisation", IEEE Trans. On Magnetics , July 2000, Vol. 36, N°4, pp 1640-1644

[24] S.Vivier, F. Gillon, M. Hecquet, P. Brochet "A Design Optimisation Manager" Compumag Evian, France July 2-5, 2001, pp 228-229