

# Electromechanical similarity question in the multidimensional traction processes of locomotive drives

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*Abstract* - From the point of view of traction tasks realization a repeatability of electromechanical traction processes is stipulated; also in consideration of motion disturbances - with reference to their level and meaning. Thus, considering the exploitation repeatability the electromechanical traction processes should be invariant. Within technological and operational reality the electromechanical traction processes should be sufficiently similar. They should be similar to each other especially in case of multidimensional electromechanical traction processes when the processes realized by equivalent sub-systems of electric locomotive drive are taken into consideration. Quantitative as well as qualitative evaluation of the electromechanical traction processes similarity in consideration of disturbances must be a result of comparative studies made on the base of the evaluation index values generated by means of always the same evaluation algorithm.

## 1. Introduction

The electric locomotive driving system is generally composed of several equivalent sub-systems. In such a case the process investigations mean the necessity to consider several electromechanical processes at the same time. A set of  $s$  electromechanical traction processes is called a  $s$ -dimensional traction process. Vehicle drive traction sub-systems - assumed to be the same and in practice being physically similar to each other - should realize similar electromechanical traction processes.

The question of evaluation of multidimensional electromechanical traction processes concerns first of all the comparative studies of their constituent processes.

## 2. Multidimensional electromechanical traction processes

In case of C0-C0 locomotive, six traction motors connected with each other may work in various systems of traction connections. In this connection a question of identification of model equations for driving system motion disturbances appears, e.g. for C0-C0 locomotive - in form of six third-order substitute differential equations for motion disturbances. Then the equation system for motion disturbances of complex locomotive drive structure may be expressed in matrix form:

$$\dot{\Phi}^* + f(\dot{\Phi}^*, \Phi^*) = F_{eq} \quad (1)$$

or as an equation system in substitute criterion form:

$$\dot{\Phi}^* + F_{kr} \cdot \Phi^* = F_{eq} \quad (2)$$

with the following components:

$$\ddot{\Phi}_k^* + c_{eq}^*(t_i) \cdot \dot{\Phi}_k^* + N_{eq}^*(t_i) \cdot \Phi_k^* = F_{eq}(u^*, i^*; t_i); \quad \text{where: } k = 1, \dots, s. \quad (3)$$

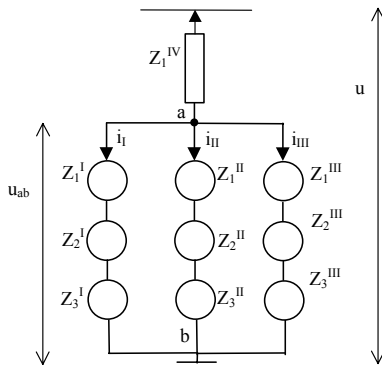


Fig. 1. Generalized arrangement of motor traction connections

The question of the multidimensionness of electromechanical traction processes following from the complex structure of electric locomotive driving system becomes important in case of necessity to determine the whole characteristics of a multidimensional electromechanical process in consideration of disturbances. Two basic cases of comparative process studies should be distinguished here. They concern as follows:

1. electromechanical traction processes of  $s$  equivalent sub-systems of locomotive driving system within the same moments  $t_i$ ,
2. electromechanical traction processes of  $i$  locomotive drive sub-system in various arrangements of moments  $t$  corresponding with subsequent transport traction processes of a particular kind.

Case 1 determines the comparison studies made as follows:

- 1a - through simultaneous expression of the observed phenomena in all the sub-systems;
- 1b - through common analysis of criterion function values when the condition of simultaneity is satisfied.

If the motion-disturbance equation system (2) satisfies the above postulates 1a and 1b, the generated runs  $F_{kr}$  of process evaluation criterion functions of  $s$  traction sub-systems will also fulfill the postulates 1a and 1b.

In case 2 - when it is not possible to compare electromechanical traction processes on the step-by-step basis, for the subsequent moments  $t_i$  a general evaluation of the processes under investigation should be taken into consideration.

### 3. Similarity of the electromechanical traction processes

The processes under evaluation are similar when similarity criteria are equal and so-called conditions of uniqueness are satisfied. In this connection, in consideration of each electrome-

chanical drive sub-system the equality of criteria in relation to the following uniqueness conditions should be assumed:

- a. the initial conditions as well as the final conditions of electromechanical traction process realization;
- b. the conditions of participation of all the traction sub-systems in the same transportation traction process;
- c. the co-ordinate systems in which the traction sub-systems are described;
- d. the physical laws that govern the system and are connected with electromechanical energy conversion;
- e. the properties of the transportation space.

Electromechanical similarity criteria in transportation traction processes should follow from the above uniqueness conditions.

The process evaluation electromechanical indices  $c_{eq}^*(t_i)$  and  $\aleph_{eq}^*(t_i)$  - equations (2) and (3) - fulfill the above uniqueness conditions, so, they may be interpreted as the criteria of electromechanical traction process similarity in multidimensional case in consideration of disturbances. Evaluation of electromechanical traction process similarity may be made by comparison of the electromechanical similarity criterion values  $c_{eq}^*(t_i)$  and  $\aleph_{eq}^*(t_i)$  of all the sub-systems of locomotive electromechanical driving system.

The process evaluation indices -  $c_{eq}^*(t_i)$  and  $\aleph_{eq}^*(t_i)$  - are determined on the base of measurement values which are, in fact, the process solution of the motion-disturbance differential equation system - the set in Fig. 2. Evaluation of multidimensional electromechanical traction processes in consideration of disturbances may be performed by comparison of their electromechanical similarity criteria in  $t_i$  moments of the process.

Taking the above said into consideration and directing the question of similarity to the process evaluation purposes the following general settlements are derived with use of the concepts of a set and power of a set:

1. The electromechanical traction processes  $\mathbf{A}=(A, o_1, \dots, o_n)$  and  $\mathbf{B}=(B, o'_1, \dots, o'_k)$  of similar traction sub-systems  $\mathbf{a}$  and  $\mathbf{b}$  are similar if  $n = k$  and for every determined  $j = 1, \dots, n$  the process operations  $o_j$  and  $o'_j$  have the same number of arguments as well as the sets  $A$  and  $B$  which run the variables of traction sub-systems and functions determined on those variables.
2. The electromechanical traction processes  $\mathbf{A}$  and  $\mathbf{B}$  are similar in consideration of disturbances if  $n = k$  and for every determined  $j$  the operations  $c_{eq_j}^*$  and  $c'_{eq_j}$  as well as  $\aleph_{eq_j}^*$  and  $\aleph'_{eq_j}$  of motion-disturbance electromechanical processes -  $\mathbf{A}^* = (A^*, c_{eq}^*, \aleph_{eq}^*)$  and  $\mathbf{B}^* = (B^*, c'_{eq}, \aleph'_{eq})$  - have the same number of variables with values belonging to the sets  $A^*$  and  $B^*$ .
3. The algorithm of electromechanical traction process evaluating generates the runs, which - in case of  $s$  driving sub-systems - determine  $s$  sets  $\{t_i, c_{eq_i}^*, \aleph_{eq_i}^*\}$ .
4. All  $s$  sets  $\{t_i, c_{eq_i}^*, \aleph_{eq_i}^*\}$  of electromechanical multidimensional process should have the following qualities:
  - a. they should have the same power i.e. they should be equinumerous sets

$$\{t_i \sim c_{eq_i}^* \sim \aleph_{eq_i}^*\}_1 \sim \dots \sim \{t_i \sim c_{eq_i}^* \sim \aleph_{eq_i}^*\}_s$$

b. they should satisfy the condition of simultaneity of all  $s$  constituent processes i.e.

$$\{t_i\}_1 = \dots = \{t_i\}_s$$

### 3.1. Quantitative evaluation of electromechanical multidimensional processes

Ordered sets of criterion function values  $c^*_{eq}, \aleph^*_{eq}$  of the  $s$ -dimensional electromechanical traction processes create tables of data  $c^*_{eq}(t_i), \aleph^*_{eq}(t_i)$  of that process with dimensions  $n \times s$ , where:  $n$  - means the number of the process moment's  $t_i$ ,  $s$  - means the number of constituent processes. Tables 3.1 are the tables the elements of which have quantitative meaning.

Tables 3.1

$c^*_{eq} =$	<table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td style="border: none;"></td> <td style="border: none;"><math>pr_i</math></td> <td style="border: none;">I</td> <td style="border: none;">II.....s</td> </tr> <tr> <td style="border: none;"><math>t_i</math></td> <td style="border: none;"></td> <td style="border: none;"></td> <td style="border: none;"></td> </tr> <tr> <td style="border: none;">1</td> <td style="border: none;"></td> <td style="border: none;"><math>c^*_{eq11}</math></td> <td style="border: none;"><math>c^*_{eq1s}</math></td> </tr> <tr> <td style="border: none;">.</td> <td style="border: none;"></td> <td style="border: none;"><math>\vdots</math></td> <td style="border: none;"><math>\vdots</math></td> </tr> <tr> <td style="border: none;">.</td> <td style="border: none;"></td> <td style="border: none;"><math>\vdots</math></td> <td style="border: none;"><math>\vdots</math></td> </tr> <tr> <td style="border: none;">.</td> <td style="border: none;"></td> <td style="border: none;"><math>\vdots</math></td> <td style="border: none;"><math>\vdots</math></td> </tr> <tr> <td style="border: none;">.</td> <td style="border: none;"></td> <td style="border: none;"><math>\vdots</math></td> <td style="border: none;"><math>\vdots</math></td> </tr> <tr> <td style="border: none;">n</td> <td style="border: none;"></td> <td style="border: none;"><math>c^*_{eqn1}</math></td> <td style="border: none;"><math>c^*_{eqns}</math></td> </tr> </table>		$pr_i$	I	II.....s	$t_i$				1		$c^*_{eq11}$	$c^*_{eq1s}$	.		$\vdots$	$\vdots$	.		$\vdots$	$\vdots$	.		$\vdots$	$\vdots$	.		$\vdots$	$\vdots$	n		$c^*_{eqn1}$	$c^*_{eqns}$
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On the base of the data tables 3.1 matrixes of distance measures  $D_i$  are formed. The objects of the distance matrix  $D_i$  represent the similarity measurements  $d(i,j)$  of the multidimensional processes. The distance matrix  $D_i$  is a symmetrical  $s \times s$  matrix formed on the base of the data tables 3.1 for the moment's  $t_i$ . In case of six sub-processes of the multidimensional process with  $s = 6$ , the distance matrix  $D_i$  has the following form

$$D_i = \begin{bmatrix} 0 & d(1,2) & \dots & d(1,6) \\ d(2,1) & 0 & \dots & \vdots \\ \vdots & \vdots & \dots & \vdots \\ d(6,1) & \dots & \dots & 0 \end{bmatrix} \quad (4)$$

Then, an element of the matrix  $D_i$  formed on the base of the data table  $c^*_{eq}$

$$d_i(3,2) = d(c^*_{eq3}, c^*_{eq2})_i \quad (5)$$

is the measure of similarity of the processes 3 and 2 within the moment  $t_i$  in accordance with accepted  $d(i,j)$  measure determination metric.

### 3.2. Qualitative evaluation of electromechanical multidimensional processes

Sometimes it may be more important to make qualitative evaluation of the electromechanical multidimensional processes. Qualification of metrical values of the indices  $c^*_{eq}(t_i)$  and  $\aleph^*_{eq}(t_i)$  may be done on the base of an established order of intensity and meaning of the  $c^*_{eq}, \aleph^*_{eq}$  values, e.g. from „bad” up to „very good” and then the assignment may be as follows:

- (bad  $\rightarrow$  b, sufficient  $\rightarrow$  s, good enough  $\rightarrow$  ge, good  $\rightarrow$  g, very good  $\rightarrow$  vg).

On this base qualitative data tables 3.2 are built.

**Tables 3.2**

$Q(c^*_{eq})=$	<table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td style="border: none;"><math>t_i</math></td> <td style="border: none;"><math>pr_i</math></td> <td style="border: none;">I</td> <td style="border: none;">II.....s</td> </tr> <tr> <td style="border: none;">1</td> <td style="border: none;">g.....vg..</td> <td style="border: none;">.....b</td> <td style="border: none;">:</td> </tr> <tr> <td style="border: none;">.</td> <td style="border: none;">:</td> <td style="border: none;">:</td> <td style="border: none;">ge.....:</td> </tr> <tr> <td style="border: none;">.</td> <td style="border: none;">:</td> <td style="border: none;">:</td> <td style="border: none;">:</td> </tr> <tr> <td style="border: none;">.</td> <td style="border: none;">:</td> <td style="border: none;">:</td> <td style="border: none;">:</td> </tr> <tr> <td style="border: none;">n</td> <td style="border: none;">s.....s.</td> <td style="border: none;">.....g</td> <td style="border: none;">:</td> </tr> </table>	$t_i$	$pr_i$	I	II.....s	1	g.....vg..	.....b	:	.	:	:	ge.....:	.	:	:	:	.	:	:	:	n	s.....s.	.....g	:
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The electromechanical multidimensional process evaluation from the point of view of similarity requires also - as in item 3.1 - determination of distant measures according to the accepted metric. Then, the data tables 3.2 are subject to statistic scaling methods according to the distribution determined for this purpose. On the base of contingency tables 3.3

**Tables 3.3**

$Q(c^*_{eq})=$	<table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td style="border: none;"><math>c^*_{eq}</math></td> <td style="border: none;"><math>b \div vg</math></td> <td style="border: none;">I</td> <td style="border: none;">II.....s</td> </tr> <tr> <td style="border: none;">1</td> <td style="border: none;">n<sub>11</sub>.....n<sub>12</sub>.....</td> <td style="border: none;">.....n<sub>1s</sub></td> <td style="border: none;">:</td> </tr> <tr> <td style="border: none;">.</td> <td style="border: none;">:</td> <td style="border: none;">:</td> <td style="border: none;">:</td> </tr> <tr> <td style="border: none;">.</td> <td style="border: none;">:</td> <td style="border: none;">:</td> <td style="border: none;">:</td> </tr> <tr> <td style="border: none;">.</td> <td style="border: none;">:</td> <td style="border: none;">:</td> <td style="border: none;">:</td> </tr> <tr> <td style="border: none;">5</td> <td style="border: none;">n<sub>51</sub>.....</td> <td style="border: none;">.....n<sub>5s</sub></td> <td style="border: none;">:</td> </tr> </table>	$c^*_{eq}$	$b \div vg$	I	II.....s	1	n <sub>11</sub> .....n <sub>12</sub> .....	.....n <sub>1s</sub>	:	.	:	:	:	.	:	:	:	.	:	:	:	5	n <sub>51</sub> .....	.....n <sub>5s</sub>	:
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the objects of which determine the frequency of event occurrence ( $b \div vg$ ) in tables  $Q(c^*_{eq})$  and  $Q(N^*_{eq})$  metrical scaling index values for particular meanings ( $b \div vg$ ) are calculated as well as metrical matrixes of over-scaled data being the base to form distance matrixes are determined.

#### 4. Summary

The existence of several equivalent sub-systems in locomotive driving systems determines the necessity to put the question of the multidimensional electromechanical traction process evaluation. The evaluation of the electric locomotive multidimensional electromechanical traction processes should enable to simultaneously consider - within a common analysis - many events and runs of many observations. This, in turn, determines the possibilities to evaluate the multidimensional electromechanical traction processes from the point of view of disturbances.

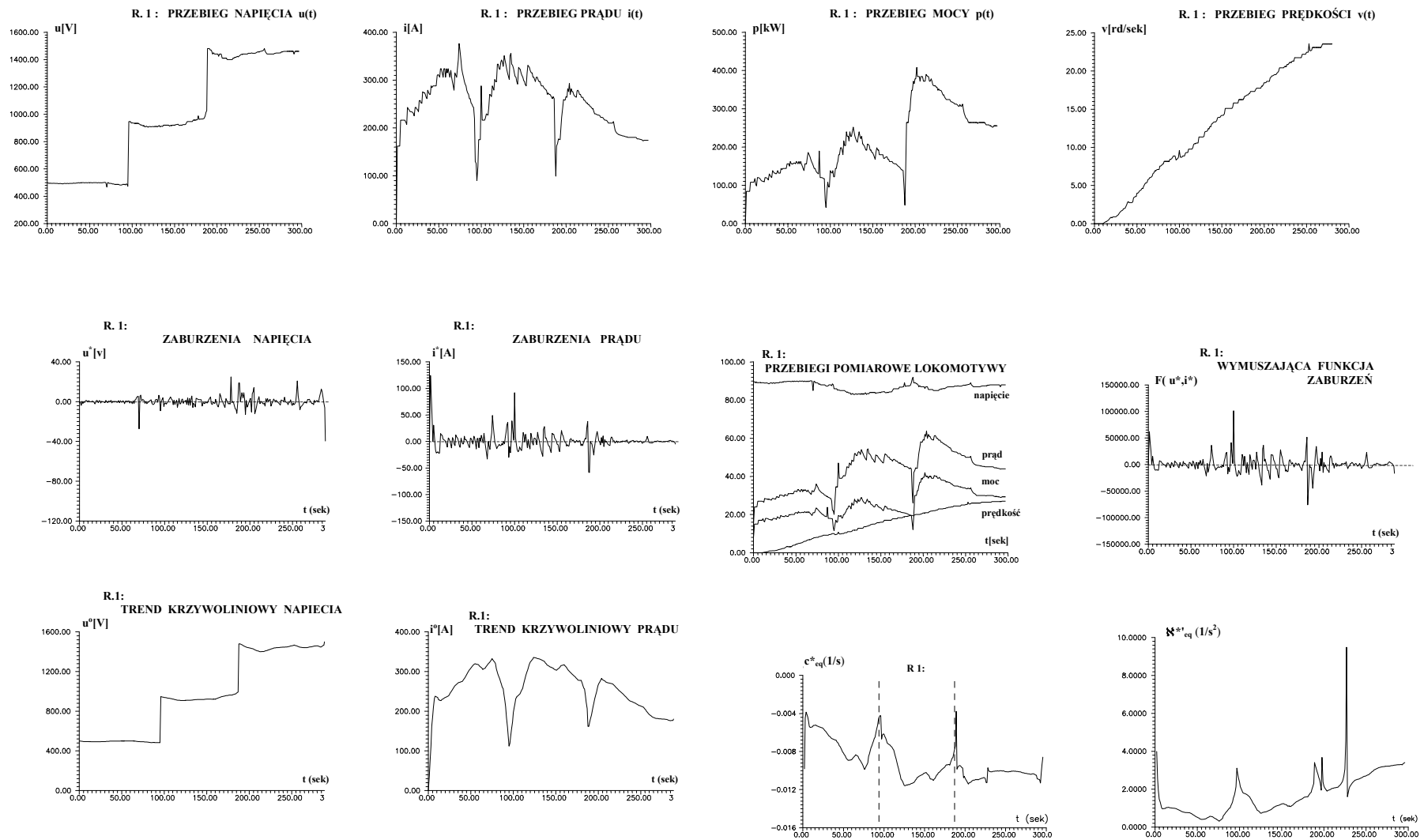


Fig. 2. The runs of the process variables and of the evaluation indices in case of the electromechanical traction processes - start up R1.

## 5. References

- [1] CAUGHEY, T.K.: Equivalent Linearization Techniques. *The Journal of the Acc. Soc. of Ann.*, vol. 35, Number 11, November 1963.
- [2] CYGAN M. Computer Aided Determination of Disturbances of Dc Locomotive Propulsive System, *SEMTRAK' 92*, Cracow University Of Technology, Cracow 1992.
- [3] CYGAN M. Computer Aided Determination of DC Basic Components of Disturbances-Input-Function of the DC Locomotive Propulsive System, *Międzynarodowa Konf. Naukowa nt. "Badania modelowe symulacji w trakcji elektrycznej"*, Tom 1, Warszawa, 18-20 listopada 1993r.
- [4] CYGAN M. Untersuchungen an Störungsmodellen elektromechanischer Lokomotivantriebe. *Diss. for Hab. degree, project.* Der Fakultät Verkehrswissenschaften der Technischen Universität Dresden, Dresden / Germany, 1993/1997.
- [5] CYGAN M. Numerical Disturbances Estimation of DC Locomotive Propulsive System in The Starting Range, *SEMTRAK' 96*, Cracow University of Technology, Kraków - Zakopane 1996.
- [6] CYGAN M. Disturbances Damping and Stiffness Modelling of DC Electromechanical Locomotive Propulsive System, Proceedings of the International Conference MV'2, Lyon/ FRANCE /, 5-6 October 1995, p. 509, *NEW ADVANCES IN MODAL SYNTHESIS OF LARGE STRUCTURES, Non-linear Damped and Non-deterministic Cases*, Edited by LOUIS JEZEQUEL, A.A. BALKEMA/ ROTTERDAM/BROOKFIELD/ 1997, ISBN 90 5410 859 2.
- [7] CYGAN M. A Computer Aided Estimation of the Starting Process Effectiveness of DC Electromechanical Locomotive Propulsive System, *3 Int. Sc. Conf. MET'97*, Warsaw, 1997.
- [8] CYGAN M. The Dynamics Estimation of the Electrical Locomotive during the Starting With Regard To Disturbances, *SEMTRAK' 98*, Cracow University of Technology, Kraków 1998.
- [9] CYGAN M. Computerised Testing Stability Changes of a DC Locomotive Drive during the Starting, *4 Intern. Scient. Conf. MET'97*, Warsaw, 1997.
- [10] CYGAN M. Monitoring Driving Wheel-Axis Behaviour of an Electrical Tractive Vehicle *SEMTRAK' 2000*, Cracow University of Technology, Cracow 2000.
- [11] CYGAN, M.: Wielowymiarowe metody zaburzeń w eksploatacyjnych badaniach elektromechanicznych napędów pojazdów szynowych. *Transport XXI Wieku, Międzynarodowa Konferencja Naukowa*; Politechnika. Warszawska, PAN, Warszawa, wrzesień 2001.
- [12] HARTUNG J., ELPELT B. *Multivariate Statistik*, Oldenburg Wissenschaftsverlag, 1999.
- [13] MASRI, S.F. & CAUGHEY, T.K.: A nonparametric identification technique for nonlinear dynamic problems. *J. Appl. Mech.*, vol. 46, pp. 433-447, June 1979.
- [14] MORRISON D. F. *Multivariate Statistical Methods*, McGraw-Hill Book Company, 1967.
- [15] RYCHLEWSKI, J.: *Wymiary i podobieństwo*. Warszawa, PWN, 1991.
- [16] SINOWJEW, A. A.: *Über mehrwertige Logik*. VEB Verlag der Wissenschaften, Berlin, 1968.

## Main designations

$c^*$  - electrodynamics damping index,

$F_{eq}(u^*, i^*)$  - disturbances input function,

$u$  - voltage momentary value,

$\aleph^*$  - electrodynamics torsion rigidity index,

$\Phi$  - mechanical angle coordinate,

Subscripts and superscripts:

$_{eq}$  - equivalent quantities,

$_k$  - means wheel in the wheel-rail system,

$_{kr}$  - concerning a criterion,

$^*$  - disturbances,

$\dot{\Phi}, \ddot{\Phi}, \ddot{\Phi}$  - subsequent coordinate time derivatives.