POLISH MARITIME RESEARCH

No 3(41) 2004 Vol 11

CONTENTS

The papers published in this issue have been reviewed by :
A. Brandowski, Prof.,D.Sc.; M. Pawłowski, D.Sc.;
M. Sperski, Assoc.Prof.,D.Sc.; J. Szantyr, Prof.,D.Sc.

PUBLISHER :

Address of Publisher & Editor's Office :

GDAŃSK UNIVERSITY
OF TECHNOLOGY
Faculty
of Ocean Engineering
& Ship Technology

ul. Narutowicza 11/12
80-952 Gdańsk, POLAND
tel.: +48 58 347 17 93
fax : +48 58 341 47 12
e-mail : sekoce@pg.gda.pl

Account number :
BANK ZACHODNI WBK S.A.
I Oddział w Gdańsku
41 1090 1098 0000 0000 0901 5569

Domestic price :
single issue : 20 zł

Prices for abroad :
single issue :
- in Europe US$ 15
- overseas US$ 20

ISSN 1233-2585

Editorial

POLISH MARITIME RESEARCH is a scientific journal of worldwide circulation. The journal appears as a quarterly four times a year. The first issue of it was published in September 1994. Its main aim is to present original, innovative scientific ideas and Research & Development achievements in the field of :

Engineering, **Computing & Technology**, **Mechanical Engineering**,

which could find applications in the broad domain of maritime economy. Hence there are published papers which concern methods of the designing, manufacturing and operating processes of such technical objects and devices as : ships, port equipment, ocean engineering units, underwater vehicles and equipment as well as harbour facilities, with accounting for marine environment protection.
The Editors of POLISH MARITIME RESEARCH make also efforts to present problems dealing with education of engineers and scientific and teaching personnel. As a rule, the basic papers are supplemented by information on conferences , important scientific events as well as cooperation in carrying out international scientific research projects.

Editorial Board

On long-term prediction of stresses in principal members of ship hull structures

Current report

Marian Bogdaniuk
Polish Register of Shipping

ABSTRACT

The paper presents a discussion on the strength criteria and load cases required in the rules of classification societies for direct FEM strength analysis of ship hull structures, which show some differences to each other. The conclusion of it is that detail studies on stress values in ship structures are necessary to improve the requirements. With this end in view an effective method for long-term prediction of stresses in ship hull structure principal members is proposed. The method is based on the concept of influence coefficients and spectral analysis of wave loads. A FEM model of the principal member system in the form of 3D frame is applied to calculate values of the influence coefficients. Next, the concept is used of correlation factors for combining characteristic long-term stress values caused by global and local loads, with combined stresses due to general bending of the ship, zone bending of the principal members and local bending of longitudinals. As an example, results of stress prediction and correlation factors calculated in some points of hull structure of a panamax bulk carrier are presented and discussed.

Key words : Rules for classification of ships, spectral analysis of wave loads on ships, dynamic stresses in ship hull structures

INTRODUCTION

Ship hull structures are usually designed according to safety standards given by classification societies in their rules for the classification and building of ships, e.g. [1] to [6]. An important issue of the rules are strength standards for the structures. Ship strength appraisal on the basis of such rules is performed according to the scheme given in Fig.1.

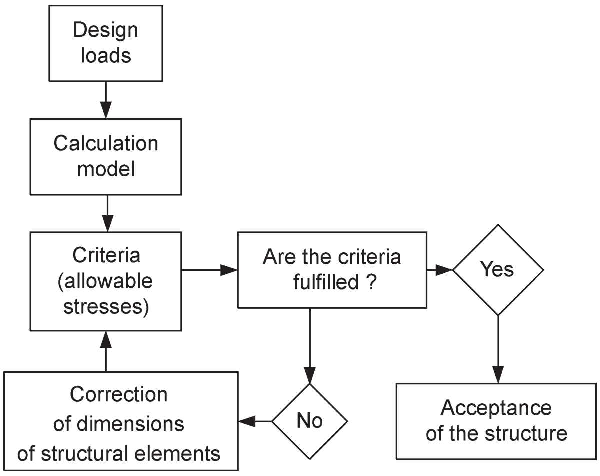


Fig.1. *Ship strength appraisal according to the rules*

There are considerable differences in the rules [1] to [6] concerning design loads, and only in very few cases the IACS member societies apply common requirements. Such case is the wave bending moment and shear force for hull general bending in vertical plane, defined in [7].

An especially important issue for ship safety is strength of hull structure longitudinal members such as double bottom girders, bottom and inner bottom stiffeners (longitudinals), etc.

In these members normal stresses due to general bending of the ship, bending of the principal members (zone strength) and local bending of the stiffeners are superimposed (Fig.2).

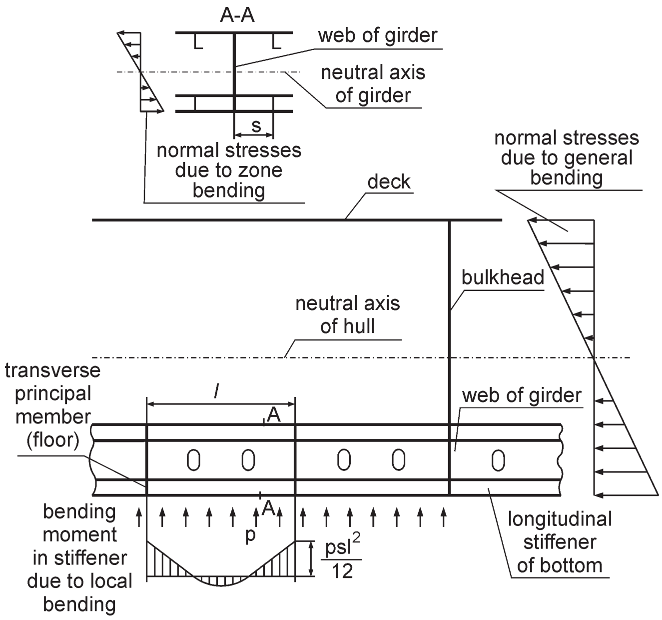


Fig.2. Normal stresses in hull structure longitudinal members

The rules [1] to [6] require that strength of the principal members is to be appraised by applying FEM model of ship hull structure module at the midship region. The FEM model should usually correspond to 3 successive holds. Some of the standard combined load cases are to be applied in the calculations. The combined load cases are composed of global load components (bending moments and shear forces at general bending of the ship, for example) and local load components (external water pressure, pressure of the cargo, etc.).

To compose the load cases, characteristic values of dynamic loads related to sailing in ocean waves, are multiplied by numbers from the range {-1;1}, called the correlation factors for combined load cases. The characteristic values of the loads are extreme values, i.e. usually those with the probability of exceedance equal to 10^{-8}.

For example, one of the combined load cases for strength appraisal of primary members, according to the requirements of DNV rules ([2 , 3]) and PRS rules ([4 , 5]), is to be composed of the dynamic loads shown in Fig.3.

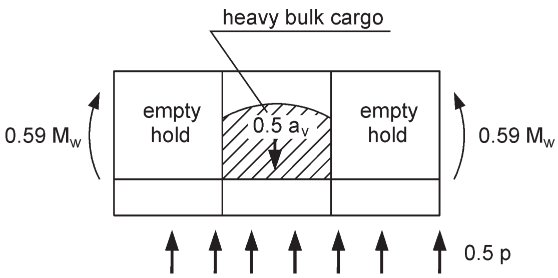


Fig.3. Dynamic load components acc. to DNV and PRS rules

So, the correlation factor for the wave bending moment M_w takes the value 0.59, for the dynamic external water pressure - the value 0.5 and for the dynamic load corresponding to vertical accelerations a_v - the value 0.5.

Quite different values of correlation factors for the load case shown in Fig.3 are required by the rules [1] and [6]. According to [1] the value 1.0 is to be applied for M_w , 0.4 - for a_v and 0.5 - for p. Whereas according to [6] the values : 0.625 for M_w , 0.0 for a_v and 0.5 for p are to be applied.

Characteristic load values in the above mentioned rules of classification societies are usually defined at different probability levels and are to be calculated by applying quite different parametric formulae. Values of the allowable stresses are also different.

These remarks allow to state that the determining of required scantlings of ship hull structure members is based on very simplified assumptions. FEM analyses required by the rules would give quite accurate values of the stresses if values of the design loads corresponded to real loads accurately. The above given short comparison of some rule requirements of the classification societies clearly shows that the required simply combined load cases must differ from the real loads.

So, a.o. in the author's opinion the searching for more accurate design loads is the most important and urgent task for development process of the rules. Therefore in PRS a research task has been undertaken to find long-term stress distributions in longitudinal members of typical ship hull structures. Results of long-term stress predictions have been compared with stress values calculated as response of the structure to the design loads defined in [4]. Analysis of the obtained results will make it possible to modify and improve the requirements concerning design loads [4].

LONG -TERM PREDICTION OF STRESSES IN LONGITUDINAL MEMBERS OF SHIP HULL STRUCTURES

For many years PRS has been developing theoretical models and software for prediction of ship's motions and loads on ship sailing in waves. The calculations based on the linear theory are performed according to the scheme shown in Fig.4.

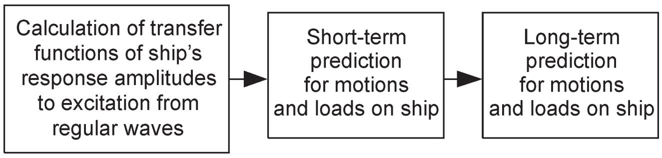


***Fig. 4.** Block diagram of the method for prediction of motions and loads on ships*

The transfer functions are calculated by using WAVE3D computer program developed in PRS. Three-dimensional water flow around the ship is taken into account.

Short-term prediction calculations take the form of spectral analysis in which Pierson-Moskowitz spectrum in terms of the significant wave height and the mean wave period, is applied [11].

The spectrum is narrow-banded and probability density function for the wave maximum values (peak values) takes the form of Rayleigh distribution, [8, 9, 10].

Long-term prediction concerns the whole life span of a ship (usually 20 years). The results of short-term predictions and statistical data in the form of joint frequency of significant wave height and mean wave period are used.The data for North Atlantic are used as usual. Detail description of PRS procedure for long-term prediction is given in [8] and [9].

The rules of classification societies should allow ship designers to assess characteristic values of stresses in the hull structure as effectively and accurately as possible. The characteristic values are those exceeded with sufficiently small probability value - e.g. 10^{-8}.

So, before formulating design loads and combined load cases for rule requirements one should start with analysis of long-term stress distributions in hull structures.

The problem is that stress values in a transverse cross-section of primary member depend significantly on the loads acting on a rather large portion of the hull structure because the primary members are mutually connected.

To make the long-term prediction of stresses in primary members as effective as possible the influence coefficients of the loads on stress values at selected points of the primary members are to be calculated before the calculations according to the scheme shown in Fig.4 start. The concept of influence coefficients is explained in Fig.5.

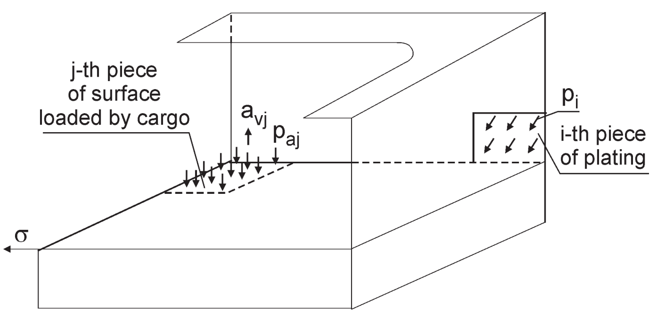


***Fig. 5.** The concept of influence coefficients*

The surface of ship hull plating loaded by the external water pressure p is divided into N_p relatively small pieces under the assumption that p = const on each piece.

Similarly the areas of inner bottom and decks loaded by pressure of cargo are divided into N_a pieces. Now the stress value σ at a point of principal member can be calculated from the formula :

$$\sigma = \sum_{i=1}^{N_p} A_i \cdot p_i + \sum_{j=1}^{N_a} B_j \cdot \overbrace{C_j \cdot a_{vj}}^{p_{aj}} \quad (1)$$

where :

N_p , N_a - as defined above
p_i - external water pressure at i-th area
a_{vj} - vertical acceleration at j-th area
A_i - the influence coefficient of p_i on σ
B_j - the influence coefficient of $C_j \cdot a_{vj}$ on σ
C_j - a coefficient to calculate p_{aj} (see Fig.5) corresponding to a_{vj} (cargo density multiplied by height of the cargo).

The values of A_i and B_j can be calculated by applying a FEM model of hull structure and assuming $p_i = 1$ or $p_{aj} = 1$ on individual pieces of external plating surface, inner bottom or deck area, and zero values at the other pieces. The FEM model in the form of a 3D frame is sufficient for the system of primary members.

To calculate transfer functions for amplitudes of σ, the amplitudes of p_i and a_{vj} (the complex numbers) calculated for the unit amplitude wave should be put into (1).

The next steps of the calculations are to be performed according to the scheme shown in Fig.4. In the double bottom girders some components of σ are superimposed (Fig.2).

The phase angles of individual components of σ for the ship on regular wave are usually different. This means that the characteristic values of the components can not be added directly. The characteristic value of σ corresponding to two components with characteristic values σ_1 and σ_2 can be written in the following form:

$$\sigma = \mathrm{Max}(\sigma_1, \sigma_2) + c \cdot \mathrm{Min}(\sigma_1, \sigma_2) \qquad (2)$$

where :

Max (σ_1 , σ_2) - the greater value of σ_1 and σ_2
Min (σ_1 , σ_2) - the smaller value of σ_1 and σ_2
c - the correlation factor for combination of σ_1 and σ_2 ; this is a number from the range {-1 ; 1}.

Such form of (2) corresponds to the above described method of combining global and local loads according to the rule requirements of classification societies, where the correlation factors are used.

RESULTS OF EXAMPLE CALCULATIONS

Long-term prediction of normal stresses σ was performed at 8 points of double bottom girders of a panamax bulk carrier, schematically shown in Fig.6.

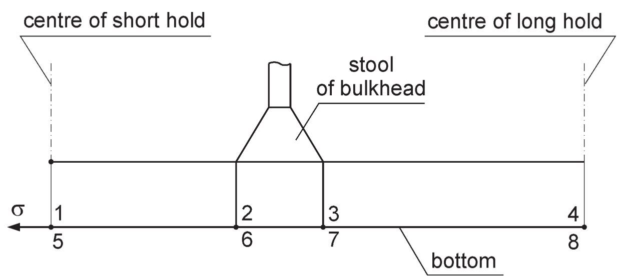


Fig. 6. *Points in bulk carrier structure where long-term prediction of normal stresses σ was performed*

Points 1 to 4 belong to the side girder 5.6 m distant from the ship's plane of symmetry. Points 5 to 8 belong to the central girder.

The influence coefficients of external water pressure and cargo inertia forces on σ were calculated by applying (1), and using the FEM model for the hull module principal members between the middles of two successive holds (see Fig.6), in the form of 3D frame.

Symmetry of frame displacements was assumed at the ends of the model, i.e. girder's transverse cross-section angles of rotation around the axis perpendicular to ship's plane of symmetry were assumed equal to zero.

Full draught of the ship d =12.2 m was assumed. The cargo inertia forces corresponding to static cargo pressure on the inner bottom equal to 100 kPa were assumed constant over the whole inner bottom area. The long-term prediction calculations were performed by applying the above described method for the following locations of the FEM model along the ship :

A - the transverse bulkhead located at the midship
B - the transverse bulkhead 50 m distant from the midship, in the bow part of the ship
C - the transverse bulkhead 50 m distant from the midship, in the aft part of the ship.

The c-factor values calculated acc. to (2) for values of σ, σ_1 and σ_2 at the probability of exceedance = 10^{-8}, at points 1 to 8 (Fig.6) are listed in the table.

Calculated values of c-factors

Point	A			B			C		
	A-1	A-2	A-3	B-1	B-2	B-3	C-1	C-2	C-3
1	0.05	0.65	0.79	0.06	0.75	0.96	0.06	0.43	0.50
5	0.10	0.77	0.81	-0.04	0.77	0.99	0.10	0.49	0.65
2	-0.06	-0.56	-0.17	-0.39	-0.37	-0.62	0.12	-0.33	0.28
6	0.12	-0.49	-0.52	-0.40	-0.39	-0.94	0.13	-0.31	0.21
3	0.05	-0.56	-0.20	-0.36	-0.43	-0.67	0.12	-0.29	0.24
7	0.12	-0.48	-0.56	-0.38	-0.42	-0.96	0.14	-0.27	0.15
4	0.09	0.71	0.75	-0.50	0.70	0.99	0.12	0.40	0.68
8	0.13	0.80	0.78	-0.50	0.76	0.99	0.14	0.48	0.66

The symbol A-1 in the table means the first case of σ_1 and σ_2 combination for the location A of the FEM model, and so on.

For the locations A, B and C of the FEM model the following combinations of normal stresses were considered :

1. $$\sigma_1 = \sigma_p \quad , \quad \sigma_2 = \sigma_a$$

where :

σ_p - the stresses in the girders, at the level of the bottom, due to zone bending by external water dynamic pressure
σ_a - as above, but caused by cargo inertia pressure.

2. $$\sigma_1 = \sigma_{pa} \quad , \quad \sigma_2 = \sigma_M$$

where :

σ_{pa} - the stresses in the girders, at the level of the bottom, due to zone bending by external water dynamic pressure and cargo inertia pressure acting together
σ_M - the stresses due to general bending of the ship in vertical plane.

3. $$\sigma_1 = \sigma_p \quad , \quad \sigma_2 = \sigma_l$$

where :

σ_p - as at p.1, but at the level of bottom stiffener flange
σ_l - the local bending stresses in the flange of bottom stiffener, where :

$$\sigma_l = \frac{psl^2}{12W}$$

s , l - see Fig. 2
W - section modulus of the stiffener with strip of the plating.

CONCLUSIONS AND FINAL REMARKS

Values of c-factors listed in the table suggest that combination of global and local stresses σ in hull structure longitudinal members is a complicated problem.

The following interesting features of such combination method can be observed :

a. The c-values for double bottom side and central girders with the same x-coordinate along the ship are almost the same, in general. This suggests that the method of stress combination in the form of (2) is reasonable.
b. There are considerable differences between c-values for individual cases of the combination (A-1, A-2, A-3, B-1, etc.). It is interesting that the values of |c| in the cases A-1 and C-1 are rather small.
c. The c-values considerably depend on x-coordinate of the points in the girders.
d. The c-values can be positive or negative. This is logical because the combination of stresses was considered. If combinations of global and local loads which cause, at a point of hull structure, stress σ equal to the result of long-term prediction based on direct calculations, were considered, then correlation factors would be the numbers of the same sign – independently of a position of a considered point.

- The paper deals with the problem of combining the stresses. According to the rules of classification societies global and local loads are to be considered to create the load cases for FEM calculations. The above listed features b. and c. of stress combination suggest that the load cases required in [1] to [6] created by means of rather simple algorithms, can lead to prediction of stress values considerably different from the real ones.
- The designing of hull structures on the basis of the rule requirements means that simplified loads are applied. The FEM calculations give quite accurate results for these loads, but calculated values of stresses can differ considerably from their real values.
- For this reason in PRS a research task aimed at formulating the load cases to be used in the rules [4] in order to obtain more accurate results of stress prediction in longitudinal hull members has been carried out. The first step of the task is to gather a set of information on stress combinations. Then the load cases which cause sufficiently accurate stress values in the structure will be searched for.

BIBLIOGRAPHY

1. American Bureau of Shipping : *Rules for building and classing Steel Vessels*. 2001, Part 5 - *Specific Vessel Types*
2. Det Norske Veritas : *Rules for Classification of Ships*, Part 3, January 2001
3. Det Norske Veritas : *Strength Analysis of Hull Structures in Bulk Carriers*. Rules for Classification. Notes No. 31.1, June 1999
4. Polski Rejestr Statków : *Rules for the classification and construction of sea-going ships*, Part II-*Hull* (in Polish). 2002
5. Polski Rejestr Statków : *Zone strength analysis of bulk carrier hull strucutre*, (in Polish). Publication No. 18/P, 1995
6. Registro Italiano Navale : *Rules for the Classification of Ships*, Part B - *Hull and Stability*, June 2000
7. International Association of Classification Societies : *Longitudinal Strength Standard*, Rev. 3, June 2003
8. Jankowski J. : *Ship Motions and Loads - formulation of Rules*. Publication No. 2 - *Classification*. Polski Rejestr Statków. Gdańsk, 1993
9. Bogdaniuk M., Jankowski J. : *An Approach to Fatigue Strength Assessment of Longitudinal Ship Hull Members*. Marine Technology Transactions, Vol. 7, Gdańsk, 1996
10. Ochi M. K.: *Ocean Waves - the Stochastic Approach*. Cambridge University Press. 1998
11. Faltinsen O. M.: *Sea Loads on Ships and Offshore Structures*. Cambridge University Press. 1990

NOMENCLATURE

a_v - vertical acceleration
A_i , B_j - influence coefficients of pressure on normal stress
c - correlation factor for normal stresses
C_j - coefficient to calculate pressure values from acceleration values
l - spacing of floors
M_w - wave bending moment
N_a - number of pieces to which the inner bottom area is divided
N_p - number of pieces to which the area of plating is divided
p - external water pressure
s - spacing of stiffeners (longitudinals)
W - section modulus of stiffener cross-section
σ - normal stress.

Acronims

DNV - Det Norske Veritas
FEM - Finite Element Method
IACS - International Association of Classification Societes
PRS - Polish Register of Shipping

CONTACT WITH THE AUTHOR

Marian Bogdaniuk, D.Sc.
Polish Register of Shipping
Al. Gen. Józefa Hallera 126
80-416, Gdańsk, POLAND
e- mail : M.Bogdaniuk@prs.pl

Conference

Scientific Meeting of Regional Group

On 25 March 2004 the Mechanical and Electric Engineering Faculty of Polish Naval University, Gdynia, hosted the first-in-the year scientific meeting of the Regional Group of the Section on Exploitation Foundations, Machine Building Committee, Polish Academy of Sciences.

During the seminar three papers were presented :

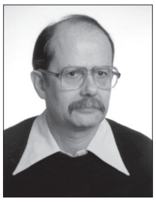
- *Recycling problems in shipbuilding - research state--of-the-art, issues to be solved* – by W. Jurczak (Polish Naval University)
- *Research on influence of fuel charge nonuniformity of combustion engine on spectrum of torsional vibration of shafting* – by St. Bruski(Polish Naval University)
- *Research on compression process to be applied to diagnosing ship piston combustion engines* – by M. Lutowicz (Polish Naval University)

After interesting discussion the participants were acquainted with a modern ship navigation bridge simulator and computer stands serving as training labotratories for navigation officers.

On hydrodynamic forces acting on the ship in large motions

Witold Błocki
Gdańsk University of Technology

ABSTRACT



Present state of mathematical description of ship dynamic non-linear behaviour is presented in this paper with a view to avoiding excessive complications in solving the problem. The non-linearity concerns first of all Froude-Krilov forces and damping forces occurring after entering ship's deck into water or those resulting from drag of bilge keels. And, to the remaining, accompanying and diffraction forces the linear extrapolation has been applied.

Key words : hydrodynamic forces, ship motions of large amplitudes, added forces

INTRODUCTION

The paper presents current state of mathematical description of ship dynamic non-linear behaviour. The subject matter concerns large-amplitude motions of ship in waves, considered within the frame of ship roll non-linear theory. The split into the accompanying forces, Froude-Krilov forces and diffraction forces is assumed still valid. The Froude-Krilov forces are calculated with taking into account changeable wetted area of ship hull surface. However to calculate the accompanying and diffraction forces a method of extrapolation of linear solutions is applied.

FROUDE-KRILOV FORCES

Froude-Krilov forces are obtained by integrating pressures in waving water not disturbed by ship's presence in it. In the non-linear theory the integration is performed over wetted surface area which results from an instantaneous position of ship hull relative to wave surface. The pressure *p* is defined by the following expression :

$$p - p_a = \rho g(z - \zeta e^{-kz}) \tag{1}$$

The Froude-Krilov resultant force $\vec{F}$ and moment $\vec{M}$ can be obtained with the use of the integration formulae :

$$\vec{F} = -\int_S \vec{n}(p - p_a)dS = -\int_V \left(\vec{i}\frac{\partial p}{\partial x} + \vec{j}\frac{\partial p}{\partial y} + \vec{k}\frac{\partial p}{\partial z} \right) dV \tag{2}$$

$$\vec{M} = -\int_S \vec{r} \times \vec{n}(p - p_a)dS =$$

$$= -\int_V \left[\vec{i}\left(y\frac{\partial p}{\partial z} - z\frac{\partial p}{\partial y} \right) + \vec{j}\left(z\frac{\partial p}{\partial x} - x\frac{\partial p}{\partial z} \right) + \vec{k}\left(x\frac{\partial p}{\partial y} - y\frac{\partial p}{\partial x} \right) \right] dV \tag{3}$$

The forces are usually calculated by means of the above presented surface integrals. In practice the wetted area is divided into a finite number of the directed area elements $\vec{n}\,\Delta S$, and integration is replaced by summation. The integration is rather troublesome as to determine the normal versor for every element ΔS is necessary. However the sufrace integrals can be replaced by scalar volume integrals in accordance with Gauss--Ostrogradski formula, which make determination of the normal versors not necessary. A way of calculation of the Froude-Krilov volume integrals was presented in [1], and realized in [2].

ADDED FORCES

The added forces acting on ship result from the forced oscillatory motion of ship in still water. They are determined by means of the formula :

$$F_i = -m_{ij}\ddot{u}_j - N_{ij}\dot{u}_j \qquad i, j = 1,2,...6 \tag{4}$$

In the formula the Einstein's convention of summation is applied. F_i , for i = 1, 2, ... 3, stands for x, y, z components of added forces, and for i = 4, 5, 6 it stands for components of moments of added forces (relative to ship's centre of gravity in the ship-fixed coordinate system). For j = 1, 2, 3 the velocities $\dot{u}_j$ stand for the components υ_x , υ_y , υ_z of the oscillation velocity of ship's centre of gravity relative to its mean location, whereas for j = 4, 5, 6 the velocities stand for the ship's angular velocity components ω_x , ω_y , ω_z (defined in the moving system). The accelerations $\ddot{u}_j$ for j = 1, 2, 3 stand for the acceleration components a_x , a_y , a_z of ship's gravity centre oscillations relative to its mean location, whereas for j = = 4, 5, 6 they stand for the components $\dot{\omega}_x$, $\dot{\omega}_y$, $\dot{\omega}_z$.

The added masses m_{ij} and damping coefficients N_{ij}, called the hydrodynamic coefficients, are constant provided the ship in question performs oscillations of constant frequency. Then they functionally depend on the oscillation frequency ω. If ship's motion is non-harmonic the added masses and damping coefficients do not have any constants and using them is senseless. In such case the hydrodynamic coefficients can be determined by using the Fourier transform method. To this end it is enough to consider the addede force component F_j resulting from j-th DOF oscillation as the Fourier transform is a linear operation.

Assuming that the ship hull oscillations occur at the amplitude u_A :

$$u_j(t) = u_A \sin \omega t \tag{5}$$

one obtains the added force :

$$F(t) = u_A \omega^2 m(\omega) \sin \omega t - u_A \omega N(\omega) \cos \omega t \tag{6}$$

The hydrodynamic force can be expressed by means of the function of response to the step excitation [3] :

$$F(t) = -\int_0^\infty r_1(\tau)\dot{u}(t-\tau)d\tau - \int_0^\infty r_2(\tau)\ddot{u}(t-\tau)d\tau \tag{7}$$

The relationship is also valid for harmonic motion. It is possible to introduce (5) to it, and next to compare it with (6) :

$$\begin{aligned} m(\omega) &= \int_0^\infty r_2(\tau)\cos \omega\tau d\tau - \frac{1}{\omega}\int_0^\infty r_1(\tau)\sin \omega\tau d\tau \\ N(\omega) &= \omega\int_0^\infty r_2(\tau)\sin \omega\tau d\tau + \int_0^\infty r_1(\tau)\cos \omega\tau d\tau \end{aligned} \tag{8}$$

Moreover, the following relationships are valid :

$$\lim_{\omega\to\infty} m(\omega) = m_\infty \qquad \lim_{\omega\to\infty} N(\omega) = 0 \tag{9}$$

The function $r_2(\tau)$ must have the following forms in order the relationships (8) to have the limits given by (9) :

$$r_2(\tau) = m_\infty \delta(\tau) \tag{10}$$

where : $\delta(\tau)$ - Dirac's delta functions.

However the function $r_1(\tau)$ must be limited and decaying along with τ increasing. Then the integral :

$$\int_0^\infty r_1(\tau)\cos \omega\tau d\tau$$

decreases along with ω increasing,with the rate of about $1/\omega$ (its exact decaying rate is unknown). On accounting for the above mentioned comments and introducing the notion : $r(\tau) \equiv$ $\equiv r_1(\tau)$, the formulae (8) are transformed into the following :

$$\begin{aligned} m(\omega) &= m_\infty - \frac{1}{\omega}\int_0^\infty r(\tau)\sin \omega\tau d\tau \\ N(\omega) &= \int_0^\infty r(\tau)\cos \omega\tau d\tau \end{aligned} \tag{11}$$

The inverse Fourier transform makes it possible to achieve an unknown response function $r(\tau)$:

$$\begin{aligned} r(\tau) &= \frac{2}{\pi}\int_0^\infty [m_\infty - m(\omega)]\omega \sin \omega\tau d\omega \\ r(\tau) &= \frac{2}{\pi}\int_0^\infty N(\omega)\cos \omega\tau d\omega \end{aligned} \tag{12}$$

From the second formula (12) one obtains the relationship for the initial value of $r(\tau)$:

$$r(\tau = 0) = \frac{2}{\pi}\int_0^\infty N(\omega)d\omega \tag{13}$$

From (12) it results that knowledge of one of the hydromechanical functions : $m(\omega)$ or $N(\omega)$ within the whole frequency range is sufficient to find the response functions $r(\tau)$. Also, it can be observed that after determination of one of the hydromechanical functions and calculation of $r(\tau)$ the other function can be determined by means of the relevant formula (11).

It is also possible to determine radiation forces in harmonic motion. On the basis of (7) the following is yielded :

$$F(t) = -m_\infty \ddot{u}(t) - \int_0^\infty r(\tau)\dot{u}(t-\tau)d\tau \tag{14}$$

The formula (14) is valid for any DOF. The integral term appearing in the formula is the so called *memory effect* which fast decays along with τ increasing. Its decaying rate depends on that of $r(\tau)$, hence it depends on the decaying rate of the damping coefficient $N(\omega)$ along with ω increasing. The above presented theory, called Cummins model, is generally related to irregular waves.

The formula (14) is generally valid for small harmonic motions of ship, performed around its equilibrium position in still water, within linear range. The linearity assumption is practically valid until the bilge does not emerge from and the deck does not immerse into water. This makes it possible to extend validity of the formula into the case of large motions, as for practical reasons it is sufficient to be limited only to large roll amplitudes and small amplitudes of the remaining motions. Different calculation procedures for added forces at large heeling angles can be met (Fig.1).

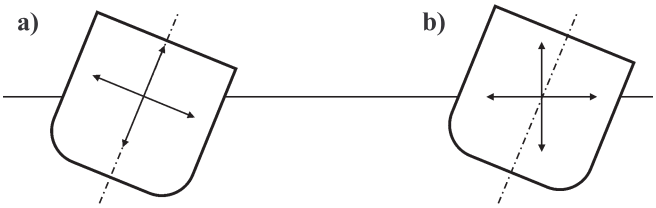


Fig. 1. *Ship motions possible to be performed around its equilibrium position in still water*

The added masses m_{ij} and the damping coefficients N_{ij} are numerically determined for the different draughts *z* and heeling angles Φ [4] :

$$\begin{aligned} m_{ij} &= m_{ij}(z,\Phi) \\ N_{ij} &= N_{ij}(z,\Phi) \end{aligned} \tag{15}$$

The forced oscillatory motion shown in Fig.1a is more favourable because the heeling angle Φ is of a negligible influence on added forces.

WATER ON THE DECK

It is assumed that the added forces determined by (14) can be also applied to heeling angles greater than the angle of deck entrance into water. However in this case the additional forces due to presence of water on the deck should be taken into account. Unfortunately in the existing computer programs the problem has been neglected though results of model tests have indicated that the additional forces due to water on the deck would be significant [7]. It is only possible to roughly estimate them as the problem is insolvable strictly.

The forces in question clearly depend on direction of motion. If the deck emerges into water they practically equal zero, and if the water flows out from the deck they become significant. The forces can be estimated by applying the momentum conservation law, and the way of their estimation is highlighted in Fig.2 [8].

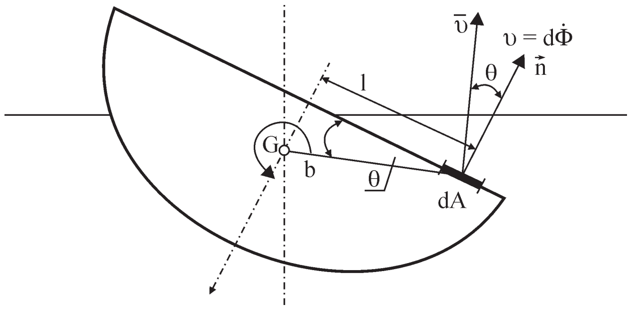


Fig. 2. Mutual relations between the deck area element dA, the normal to the deck, $\vec{n}$, and the deck area element velocity $\overline{\upsilon}$

The momentum conservation law was used along the direction $\vec{n}$ for the water head over the deck area element dA. The water is subject to a change of momentum with time, which, according to 2nd principle of dynamics, is equal to the deck reaction R_n :

$$\frac{d}{dt}(m\upsilon_n) = R_n \tag{16}$$

or :

$$\frac{dm}{dt}\upsilon_n + m\frac{d\upsilon_n}{dt} = R_n \tag{17}$$

The continuity equation for the water head over the deck area element dA subject to momentum change, can be written as follows :

$$\frac{dm}{dt} = \rho\upsilon_n dA \tag{18}$$

Taking into account the following auxiliary relationships (Fig. 2) :

$$\upsilon = b\,\dot{\Phi} \quad , \quad \upsilon_n = \upsilon \cos\theta$$

$$\upsilon_n = b\,\dot{\Phi}\cos\theta = l\dot{\Phi} \quad , \quad \frac{d\upsilon_n}{dt} = l\ddot{\Phi}$$

one can transform (17) into the following :

$$R_n = \rho\dot{\Phi}^2 l^2 dA + \rho\ddot{\Phi} l dV \tag{19}$$

The equation (19) expresses a unit force acting on the ship deck during water running off the deck. The elementary moment of the force R_n relative to x - axis is expressed by the following relationship:

$$dK = \rho\dot{\Phi}^2 l^3 dA + \rho\ddot{\Phi} l^2 dV \tag{20}$$

The additional moment due to water running off the deck can be obtained by integrating (20) over the wetted deck area A and the volume of the water appearing on the deck, V :

$$K = \dot{\Phi}^2\rho\int_A l^3 dA + \ddot{\Phi}\rho\int_V l^2 dV \tag{21}$$

The approximate formula (21) for the additional moment due to water running off the deck can be presented as the sum of two elements : the damping moment and the added mass moment :

$$K = I_3\dot{\Phi}^2 + i_X\ddot{\Phi} \tag{22}$$

where :

I_3 - 3rd order moment of the wetted deck area
i_x - mass inertia moment of the water appearing on the deck, respective to the ship central axis x.

DIFFRACTION FORCES

The diffraction forces result from hull-induced disturbances of pressure distribution in waving water. In the linear theory the forces in question are calculated on the basis of wave action on motionless ship. The assumption is valid in the case of small motions only. Nevertheless it is also extended into large motions. The diffraction forces acting on a ship in irregular waves at large motion amplitudes are determined by superposing the forces resulting from particular harmonic components. It results from the assumption, common for added and diffraction forces, in which the ship is further considered, despite large amplitudes of motion, as a linear object respective to those forces. The assumption makes calculations much simpler as it allows for using the characteristics of the ship in upright position, which are independent on time and instantaneous positions of the ship. Otherwise it would be necessary to determine them in every time step and for every instantaneous wetted area of hull surface, which is a very difficult task.

If to denote, by F_{Dsij} and F_{Dcij}, respectively the sinusoidal and cosinusoidal parts of the amplitude of the diffraction force associated with i-th DOF and resulting from j-th harmonic component, then the generalized diffraction force F_{Di} can be expressed as follows:

$$F_{Di}(t) = \sum_j \begin{bmatrix} F_{Dcij}\cos(\omega_{Ej}t - \varepsilon_j) + \\ + F_{Dsij}\sin(\omega_{Ej}t - \varepsilon_j) \end{bmatrix} \tag{23}$$

where the indicated summation is performed over all harmonic components.

CONCLUSIONS

- The radiation and diffraction forces are smaller than Froude-Krilov ones approximately by one order of magnitude [5]; they influence first of all the phase shift angles between waves and ship motions, and – to a much smaller degree – amplitudes of the motions [6].
- On the basis of the subject-matter literature it can be stated that the differences between the linear theory of ship motions and the non-linear one are not large, and that they do not significantly influence solutions of ship motion equations if only motions of water relative to ship are assumed to occur within the range of ship's sides.

NOMENCLATURE

a - acceleration component of ship gravity centre oscilation
A - ship deck wetted area
b - distance between the deck surface area element dA and the ship gravity centre G
$\vec{F}$ - hydrodynamic Froude-Krilov resultant force acting on ship hull
F_i - components of added forces
F_{Di} - diffraction force component
F_{Dcij} - cosinusoidal component of diffraction force amplitude
F_{Dsij} - sinusoidal component of diffraction force amplitude
g - gravity acceleration
$\vec{i}, \vec{j}, \vec{k}$ - versors
k - wave number
K - additional moment resulting from the water running off the deck
l - distance between the deck surface area element dA and the ship plane of symmetry
m - mass of water on the ship deck surface element dA
m_{ij} - ship added masses
m_∞ - added mass corresponding to the angular frequency ω tending to infinity

$\vec{M}$ - hydrodynamic Froude-Krilov resultant moment acting on ship hull
$\vec{n}$ - unit vector normal to ship surface
N_{ij} - ship damping coefficients
p - pressure inside wave
p_a - atmospheric pressure
r - response to the step excitation
$\vec{r}$ - tracing vector
R_n - unit force acting on the deck surface area element dA
S - instantaneous ship hull surface wetted area closed by free surface of wave
t - time
u - oscillation of ship gravity centre
u_A - ship hull oscillation amplitude
u_j - oscillation of ship gravity centre in the j-th direction respective to its mean position
υ - velocity of the deck surface area element dA
υ_n - projection of the velocity $\bar{\upsilon}$ onto direction of the normal $\vec{n}$
V - instantaneous volume of immersed part of ship hull; volume of water on the deck
x, y, z - spatial coordinates
ε_j - phase shift angle of harmonic component
ξ - wave profile ordinate
θ - angle between vectors of the velocity $\bar{\upsilon}$ and the normal $\vec{n}$
ρ - water density
τ - time shift angle
Φ - ship heeling angle
$\dot{\Phi}, \ddot{\Phi}$ - ship roll angular velocity and acceleration, respectively
ω - oscillation frequency
ω_E - encounter frequency of the ship

Acronims

CTO - Ship Design and Research Centre
DOF - degrees of freedom
ISC - International Shipbuilding Conference
PRS - Polish Register of Shipping

BIBLIOGRAPHY

1. Pawłowski, M.: *A nonlinear model of ship roll in irregular waves* (in Polish). Technical report of PRS, No. 33/99, Gdańsk, February 1999
2. Laskowski, A.: *Numerical modelling of damaged ro-ro ship's motions in waves* (in Polish), doctorate thesis. Gdańsk University of Technology. 2002; also : *Numerical modelling of damaged ro-ro ship's motions in waves*. Polish Maritime Research, No. 3. Gdańsk, 2003
3. Cummins W.E.: *The impulse response function and ship motions*. Schiffstechnik, Vol. 9, No. 47, June 1962
4. Dudziak J.: *Oscillations and wave loads of ship in head seas* (in Polish). Scientific bulletin CTO, No. B-088, Gdańsk, December 1999
5. Dudziak J., Pawłowski M.: *A simplified approach to time domain analysis of ship behaviour in head irregular seas*. ISC'98 Proceedings, Section B. St. Petersburg, 1998
6. Dudziak J.: *Computer simulation of ship roll in waves* (in Polish). Scientific bulletin CTO, No. B-061, Gdańsk, December 1993
7. Błocki W.: *Model tests of parametric roll of ship* (in Polish). Report of Supporting Enterprise of Ship Research Institute, Technical University of Gdańsk, No. 558, Gdańsk, 1975
8. Błocki W.: *Hydrodynamic moments due to water on ship deck*. Computer Assisted Mechanics and Engineering Sciences, No. 5. Polish Academy of Sciences. Warszawa, 1998

CONTACT WITH THE AUTHOR

Witold Błocki, D.Sc.
Faculty of Ocean Engineering
and Ship Technology,
Gdańsk University of Technology
80-952 Gdańsk, POLAND
Narutowicza 11/12
e-mail : sekoce@pg.gda.pl

Conference

10th Jubilee Seminar

On 18 February 2004 the 10th, successive Seminar on

Drives and Control Systems

was held in Gdańsk. The series of such seminars accompanying the Fairs of Manufacturers, Subcontractors and Providers of Power Units and Control Systems, have been each year organized by Gdańsk University of Technology since 1995. This has been a good occasion for direct mutual contacts of representatives of scientific centres working at development of this engineering branch and the manufacturers and dealers of such technical devices, which next brings about novel solutions based on results of R&D projects.

As usual, the Seminar was devoted to the problems of the following areas :

- drives and control systems of machines
- automation and dynamic behaviour of driving systems for shipbuilding and power industries
- electronic devices applicable to driving and control systems
- automation of electric drives
- applications of hydraulic and pneumatic drives
- applications of control and signal processing methods.

The problems were discussed during three sessions :

A - *Mechanical, hydraulic and pneumatic drives* (12 papers)
B - *Automation of electric drives* (6 papers)
C - *Applications of control and signal processing methods* (7 papers),

and, during one poster session another 22 topics were presented.

Moreover, the Seminar was accommpanied with the meetings organized in the frame of the workshops devoted to the following topics :

- I - *Control techniques for linear electro-hydraulical servodrives*. (Technical University of Poznań)
- II - *Advanced simulation techniques for converter drives. Application of Tcad package*. (Gdańsk University of Technology)
- III - *Automation of ship electric power system*. (Gdynia Maritime University)
- IV - *Electronic microsystems - their design, diagnostics and integration*. (Gdańsk University of Technology in cooperation with TASC company).

STG Conference in Szczecin

From 1 to 4 June 2004 a group of members of the German Society of Naval Architects and Marine Engineers (Schiffbautechnische Gesselschaft - STG) took part in its yearly summer meeting held abroad. An important event of the meeting has always been a scientific conference. This year it was held on 2 and 3 June at Szczecin Maritime University, the local host of the conference.

The conference was carried out in four sessions :

- ☆ Inland Shipping; Strength – 3 papers
- ☆ Hydrodynamics – 4 papers
- ☆ Ship Design and Ship Safety – 3 papers
- ☆ Ship Propulsion – 8 papers.

During the last session 4 papers were presented by scientific workers of Szczecin Maritime University :

- ➲ *An experimental investigation of influence of fuel injection timing on exhaust emission of marine low-speed engines* – by T. Borkowski, D.Sc.
- ➲ *Examinations of structure and selected chemical properties of the sludge produced during cleaning process of residual fuel, lubrication oil and bilge water* by P. Rajewski, D.Sc.
- ➲ *Design and facilities of a testing stand of oily ship water cleaning process* – by J. Listewnik, Assoc. Prof., D.Sc., A. Wiewióra, D.Sc.
- ➲ *Identification of marine generators in service and maintenance* – by P. Bielawski, D.Sc.

The Conference participants had the opportunity of visiting Szczecin including its historical monuments and a shipyard, as well as to take part in tours to neihbouring small historical towns and health resorts.

Moreover the host of the Conference presented some of the University's research facilities and research projects, namely:

- ❍ Newly developed homogeniser and its performance tests
- ❍ EU- funded research project on machinery maintenance
- ❍ Bilge water deoiling test stand.

A conference under sail

From 28 March to 3 April 2004 1st Domestic Scientific Conference, an original conference on :

Scientific and technical problems of professional sailing

was held on board the sail ship POGORIA during her voyage on the route :
Genoa - Elba - Bonifacio - Nice - - Monaco - Portoferraio - Genoa.

It was organized by Faculty of Mechanical Engineering of Gdynia Maritime University, Faculty of Mechanics, Energy and Aeronautics and Faculty of Motor cars and Heavy Machinery of Warsaw University of Technology, and had on its agenda presentation of 12 papers split into 3 topical groups :

Measuring and testing

- ❖ *On possible assessment of a yacht in natural conditions* by Zb. Dąbrowski (Warsaw University of Technology) (an invited paper)
- ❖ *Multi-channel data acquisition and processing system for steering the sailing boat by means of TCP/IP* by M. Pilarski (Warsaw University of Technology)
- ❖ *Design of a system for determination of polar characteristics of yacht speed* – by P. Kłopotowski (Warsaw University of Technology)
- ❖ *Application of MLS method in testing acoustic qualities of small compartments on yachts* – by W. Batko, T. Wszołek, W. Wszołek (Mining - Metalurgical Academy)
- ❖ *Vibration propagation measure as an index of yacht mast profile weakening* (abstract) – by G. Klekot (Warsaw University of Technology).

Novel materials and engineering processes

- ✸ *Some problems of manufacturing and operation of composite masts* (an invited paper) – by W. Skórski (Warsaw University of Technology)
- ✸ *Dynamic features of modern composite masts* by P. Dełuszkiewicz, J. Dziurdź (Warsaw University of Technology)
- ✸ *Problems of modelling the polimer - structure materials* by M. Dudziak
- ✸ *Problems of transferring the concentrated forces into composite structures* – by M. Rodzewicz (Warsaw University of Technology)
- ✸ *Problems of application of metal sandwich structures for elements of sea-going ships* (abstract) – by J. Kozak (Gdańsk University of Technology).

Self - steering gear and autopilot

- ❈ *Design of a self-steering gear having an actively controlled vane-fixing angle* – by P. Górecki, B. Stankiewicz, P. Szajkowski (Warsaw University of Technology)
- ❈ *A simple yacht autopilot controlled by means of electronic compass* – by P. Kłopotowski, St. Misiaszek (Warsaw University of Technology).

Photo: C. Spigarski

Roll response of ship fitted with passive stabilizing tank

Ludwik Balcer
Gdańsk University of Technology

ABSTRACT

Physical and mathematical models of roll motions of a ship equipped with a roll stabilizing tank of working liquid free surface, is presented. Elaboration of the physical model was based on the idea of two mutually coupled mathematical pendulae. On the basis of the physical model, motion equations of the ship with the tank were determined and solved. A way of using the achieved solutions is shown, as well as calculation formulae for coefficients of the motion equations, directly related to the main parameters of the ship and tank, are presented. Such form of the coefficients enhances possiblility of application of the equations and their solutions in ship design practice. Some examples of the use of the solutions for analysis of stabilizing effectiveness of a designed tank for a given ship, are also attached. Moreover, guidelines for correct design of the stabilizing tanks having free surface of liquid, based on the proposed physical model of the ship-tank system, are offerred. It is also indicated that on the basis of the presented results it would be possible to search for ways to make operation of the stabilizing tanks in question more effective.

Key words : seaworthiness, ship safety, ship hydromechanics, stability of floating units

INTRODUCTION

Out of the oscillation motions performed by a ship in waves the side roll (heeling) undergoes stabilization most often. The stabilization means first of all a limitation of roll amplitude values, sometimes associated with increasing its period. The stabilizing tanks of free surface of liquid, which are further considered in this paper, are simple devices intended for limiting roll motions.

A disadvantage of the existing methods describing work of stabilizing tanks is their low usefulness for designing. The equations applied in the methods are rather complicated [8, 10, 12, 13 , 14] as they are usually aimed at correct describing real motions of ship fitted with tank, and not at achieving practical design recommendations.

In this paper is presented such physical model of the ship with stabilizing tank under rolling and mathematical model based on it, as to obtain motion equations of a possibly simple form more useful for formulating important design recommendations. Coefficients of such equations are directly associated with ship and tank parameters. The obtained form of solutions makes it possible to get access to these parameters of ship and tank whose selection is crucial for ensuring expected stabilizing effects.

The simplifications introduced to obtain an appropriate form of the equations and their solutions do not impair to an evident degree the quality of the description of ship motions, based on them.

FREE - SURFACE STABILIZING TANKS

The basic types of free-surface stabilizing tanks presented in Fig.1 , 2, and Fig.3 and 4 exemplify their possible location on ships. An appropriate adjustment of natural period of motions of liquid contained in tank to that of rolling ship or of wave acting on ship, is of a dominant influence on stabilizing effect. The way of changing the motion period of liquid in such tanks consists in changing level of the liquid, which also leads to changing its amount.

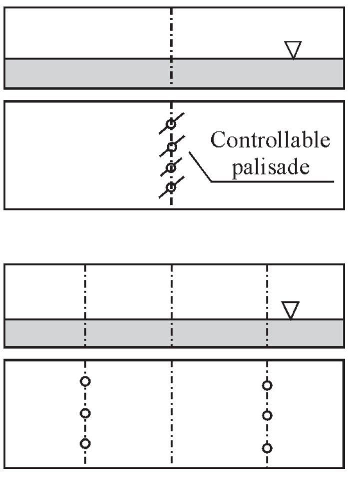
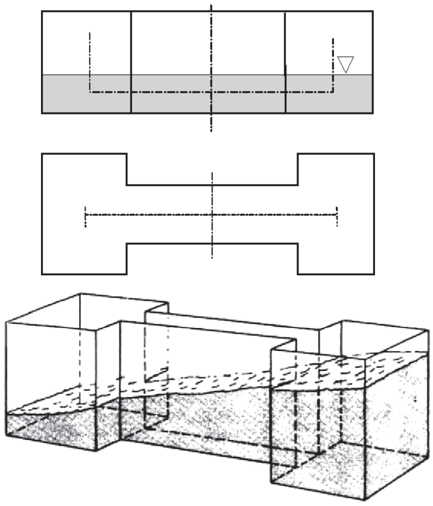
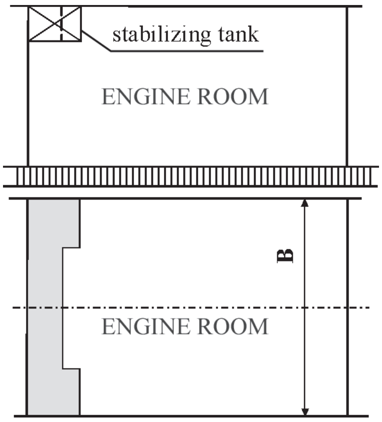


Fig. 1. *Types of cubicoidal stabilizing tanks [16]*



Rys. 2. *"Flume" stabilizing tank*



Rys. 3. *Location of "Flume" tank onboard a dry cargo ship*

Rys. 4. *Stabilizing tank in the form of container installed on deck of a fishing ship [2]*

Behaviour of a ship with stabilizing tank can be limited to consideration of motions of the system of two degrees of freedom (DOF), namely : for ship – side rolling angles, for tank – – translations of centre of mass of the liquid contained in it. The liquid mass centre translation can be represented by the average slope angle of its surface relative to ship, against its initial position. Hence description of motions of the ship-tank system amounts to the known problem of motion of a two-DOF system, whose solutions are based on the Lagrange 2nd kind equations. However in order to form such equations a "physical" model of the ship-tank system, i.e. physical representation of the ship, of the tank and their mutual coupling should be first determined. A form of external excitation of motions of the system, due to wave action on ship, should be also given.

PHYSICAL MODEL OF SHIP - TANK SYSTEM

A rolling ship is usually represented by a physical pendulum of an inclination moment dependent on ship's metacentric height. The mathematical pendulum is the simplest and lightest out of possible physical ones of a given inclination moment and period. The system of two mutually coupled pendulae is a known mechanical analogy of the two-DOF oscillating systems. It was assumed that the ship with stabilizing tank can be represented by the system of two mathematical pendulae appropriately selected and mutually coupled. The main oscillation axis of the pendulae corresponds to location of ship's rolling axis. The location of ship's rolling axis was assumed constant, known and not necessarily identical with that of ship's mass centre [1]. A schematic diagram of the physical model is shown in Fig.7, whereas Fig.5 and 6 highlight its association with the main parameters of ship and tank. The necessary modelling principles according to which the elements of the pendulae and their coupling parameters have been determined, are given by the formulae (1) below. The main coupling parameter of the pendulae is the distance between their rotation axes; i.e. the segment $\overline{O'_S m'}$ in Fig.7. The point O'_S represents the location of the ship's rolling axis, O_S.

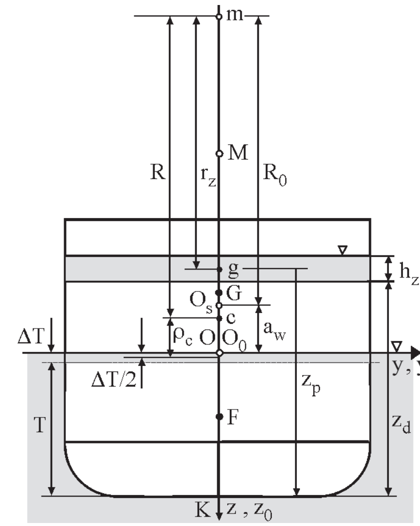


Fig. 5. *Ship with stabilizing tank in the upright position*

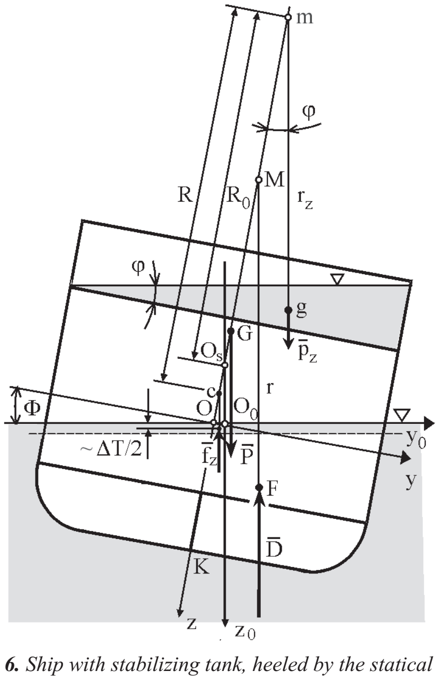
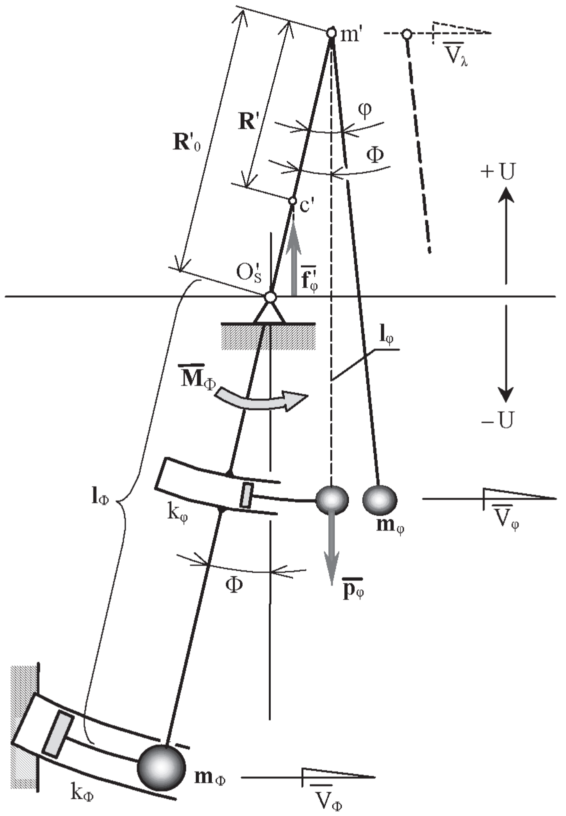


Fig. 6. *Ship with stabilizing tank, heeled by the statical angle* φ



Modelling principles

$$l_\Phi = \frac{\tau_\Phi^2 g}{4\pi^2} \qquad l_\varphi = \frac{\tau_\varphi^2 g}{4\pi^2}$$

$$m_\Phi = \frac{m_s h_s}{l_\Phi} \qquad m_\varphi = \frac{m_z r_z}{l_\varphi}$$

$$R' = R\frac{m_z}{m_\varphi} \qquad R'_0 = R_0\sqrt{\frac{l_\varphi}{r_z}}$$

$$\overline{f'_\varphi} = -m_\varphi g$$

$$M_\Phi = Dh\alpha_m \sin\omega_\lambda t$$

(1)

Fig. 7. *Physical model of a ship with stabilizing tank*

Modelling principles

The system of pendulae shown in Fig.7 represents a physical model of a ship with stabilizing tank having free surface of liquid. The model can be deemed correct if statical and dynamical response of the system to external excitation moment is the same as that for the real ship with tank. For the pendulum of ship and tank the following can be written :

$$l_\Phi = \frac{r_{sm}^2}{h_s} \quad \text{or} \quad l_\Phi = \frac{\tau_\Phi^2 g}{4\pi^2} \tag{2}$$

$$l_\varphi = \frac{r_{zm}^2}{r_z} \quad \text{or} \quad l_\varphi = \frac{\tau_\varphi^2 g}{4\pi^2} \tag{3}$$

where :

- τ_Φ - period of natural oscillation of ship's mathematical pendulum, and of ship itself
- l_Φ - length of ship mathematical pendulum
- r_{sm} - radius of ship mass inertia
- r_{zm} - radius of mass inertia of liquid in tank
- r_z - metacentric radius of tank
- g - acceleration of gravity
- h_s - initial metacentric height of ship without tank, $\overline{GM_s}$
- τ_φ - period of natural oscillation of tank mathematical pendulum, and of motion of liquid in the tank
- l_φ - length of tank mathematical pendulum.

The static condition can be expressed by the relationships (4) and (5). The expressions (4) concern the equality of the moments due to heeling for the ship itself, the tank itself and, respectively, for the pendulae being their physical models. By using them the value of the mass m_Φ - for the ship mathematical pendulum, and of the mass m_φ - for the tank pendulum, can be obtained :

$$m_s h_s = m_\Phi l_\Phi \;, \quad m_\Phi = \frac{m_s h_s}{l_\Phi}$$
$$m_z r_z = m_\varphi l_\varphi \;, \quad m_\varphi = \frac{m_z r_z}{l_\varphi} \tag{4}$$

where :

m_s - ship mass (without tank)
m_z - tank mass, i.e. mass of liquid contained in the tank.

The total static condition has to account for similarity of the righting moments for the heeled ship with tank and for the system of pendulae inclined by the same angle. Rigidity of the ship with tank and of the pendulae can be expressed by using the quantities shown in Fig.5, Fig.6 and Fig.7, which provides the following equality :

$$m_s h_s - m_z R = m_\Phi l_\Phi - m_\varphi R' = (m_s + m_z)h - m_z r_z \tag{5}$$

where :

- $R = \overline{mc}$ - the distance between the tank metacentre and the metacentre of the ship's immersed volume increment ΔV
- $R' = \overline{m'c'}$ - the distance, in the physical model , which corresponds to R of the ship
- h - initial metacentric height of the ship with tank without any correction for liquid free surface
- $m_z r_z = i_x \rho$ - correction for free surface of liquid contained in tank.

To the system of pendulae the same moment due to water in tank must be applied as that applied to the ship. Knowing that : $m_s h_s = m_\Phi l_\Phi$ one obtains, acc.to (5) : $m_z R = m_\varphi R'$. It corresponds to the moment of couple of forces. For the ship with tank these are the forces : $\overline{p}_z = \overline{f}_z = m_z \overline{g}$ (Fig.6), and respectively for the system of pendulae $\overline{p_\varphi} = \overline{f_\varphi'} = m_\varphi \overline{g}$ (Fig.7).

Hence in the physical model the value of R' amounts to :

$$R' = R\,\frac{m_z}{m_\varphi} \quad , \quad R = z_p + r_z - T - \Delta T/2 - \rho_c \qquad (6)$$
$$\rho_c = \Delta I_{wx} / \Delta V$$

where :

ρ_c - metacentric radius of displacement layer of ΔT in thickness [7]

ΔI_{wx} - increment of transverse inertia moment of waterplane area for a given ΔT.

The dynamic condition first of all consists in the equality of the inertia moment of the ship and that of its pendulum, as well as the mass inertia moment of liquid in the tank and that of its pendulum. Moreover has to be fulfilled the equality of kinetic energy of the ship with tank and that of the system of pendulae when the same roll inducing moments are applied to them.

Hence the following is obtained for the ship and tank :

$$m_s r_{sm}^2 = m_\Phi l_\Phi^2 \quad , \quad m_z r_{zm}^2 = m_\varphi l_\varphi^2 \qquad (7)$$

All the elements in that expression have been already determined by the statical condition. By means of the expressions (2), (3) and (4) it is easy to check that the above given relationship is satisfied. Kinetic energy of the ship with tank will be equal to that of the system of pendulae, if the following relationship is satisfied :

$$m_s r_{sm}^2 + m_z R_0^2 + m_z r_{zm}^2 = m_\Phi l_\Phi^2 + m_\varphi R_0'^2 + m_\varphi l_\varphi^2$$

where :

R_0 - the distance between tank's metacentre and ship's rolling axis, $\overline{mO_s}$

R_0' - the distance between the suspension point of the tank mathematical pendulum and the oscillation axis of the system of pendulae, $\overline{m'O'_s}$.

From (7) it results: $m_z R_0^2 = m_\varphi R_0'^2$, hence, after taking into account (4), one obtains the following expression for R_0' :

$$R_0' = R_0 \sqrt{\frac{l_\varphi}{r_z}} \quad \text{or :} \quad R_0' = R_0 \frac{r_{zm}}{r_z} \qquad (8)$$
$$R_0 = z_p + r_z - T - \Delta T + a_w$$

The quantity a_w appearing in (8) determines the distance of the rolling axis from the waterplane (Fig. 5) [1]. It concerns the draft T and the mass centre location coordinate z_G for the ship with "frozen" liquid in the tank.

General information on effects of operation of a tank selected for a given ship can be achieved on the basis of the roll trasfer function of ship with tank. Therefore the roll is induced by the heeling moment resulting from the action of regular plane beam wave on the motionless ship. Wave frequencies to be selected should properly cover the whole range of encounter frequencies possible to occur in service of the given ship. The roll inducing moment due to regular plane wave is of the following form [3,11] :

$$M_\lambda = D\,h\,\alpha_m \sin \omega_\lambda t \quad , \quad \alpha_m = \kappa_B\,\kappa_T\,\alpha_\lambda \qquad (9)$$

where :

M_λ - ship roll inducing moment

D - ship buoyance force

h - ship metacentric height

α_λ , α_m - amplitude and effective amplitude of wave slope angle, respectively

ω_λ - wave frequency

t - time

κ_B , κ_T - wave slope angle corrective coefficients dependent on B/λ i T/λ , where λ stands for wave length.

In the expression (9) the buoyance force D and metacentric height *h* concern the ship with tank. It may take also another form if influence of sway on motions of liquid in the tank is accounted for. To take into account the sway it was assumed that it influence solely motions of liquid in the tank [15], and is equivalent to the horizontal component of orbital motion of the ship in real waves. The radius of the motion is equal to a half of wave height associated with the amplitude of its effective slope angle, α_m. And, for the horizontal oscillations the following is valid :

$$y_\lambda(t) = -\frac{H_m}{2}\alpha_\lambda(t) \quad , \quad \alpha_\lambda(t) = \alpha_m \sin\omega_\lambda t$$

therefore :

$$\dot{y}_\lambda(t) \equiv V_\lambda = -\frac{g}{\omega_\lambda}\alpha_m \cos\omega_\lambda t \qquad (10)$$

where :

$\dot{y}_\lambda$ - ship sway

H_m - wave height corresponding to the effective slope angle α_m

V_λ - horizontal transverse component of ship sway-induced velocity.

In the physical model the accounting for sway is equivalent to the applying of the horizontal oscillations V_λ, to the pivoting axis of the tank pendulum (Fig.7). The so obtained motion equations of the tank mathematical pendulum, at neglecting the damping, are as follows :

$$\ddot{\varphi} + \frac{g}{l_\varphi}\varphi = \frac{H_m}{2 l_\varphi}\omega_\lambda^2 \sin\omega_\lambda t$$
$$\ddot{\varphi} + \omega_\varphi^2 \varphi = \omega_\varphi^2 \alpha_m \sin\omega_\lambda t \qquad (11)$$

It can be observed that instead to take into account ship sway in the physical model it is possible to apply, to the tank pendulum, the excitation moment whose normalized form is given by the right-hand side of the relationship (11). The same value of the moment was used in compliance with the Watanabe's method, in [15].

MATHEMATICAL MODEL

The mathematical model is equivalent to the set of motion equations of ship with stabilizing tank, achieved on the basis of the above presented physical model. A searched solution is the roll transfer function further used for analyzing the behaviour of ship with tank. Solutions are searched for within the frame of linear approach.

Derivation of motion equations

The Lagrange 2nd kind equations are used for derivation of motion equations :

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_j}\right) - \frac{\partial E}{\partial q_j} + \frac{\partial \theta}{\partial \dot{q}_j} + \frac{\partial U}{\partial q_j} = M_{q_j} \qquad (12)$$

In the equation the angle Φ - for the ship's pendulum and the angle φ - for the tank's pendulum corresponds , in the physical model, to the quantities q_j , namely q_1 and q_2 , respectively [9, 12, 13].

Particular kinds of energy considered in the physical model are the following : E - kinetic energy, U - potential energy and Θ - dissipation energy. In Fig.7, the horizontal line passing through the axis of the system of pendulae represents the plane of zero- value potential energy of the system.

The kinetic energy of the system of pendulae is of the form :

$$E = m_\Phi l_\Phi^2 \frac{\dot\Phi^2}{2} + m_\varphi (l_\varphi - R_0')^2 \frac{\dot\Phi^2}{2} + m_\varphi (l_\varphi - R_0') l_\varphi \dot\Phi \dot\varphi + m_\varphi (l_\varphi - R_0') \dot\Phi V_\lambda + m_\varphi l_\lambda \dot\varphi V_\lambda + m_\varphi l_\varphi^2 \frac{\dot\varphi^2}{2} + m_\varphi \frac{V_\lambda^2}{2} \qquad (13)$$

The potential and dissipation energies are as follows :

$$U = [m_\Phi g l_\Phi - m_\varphi g (R' - l_\varphi)] \frac{\Phi^2}{2} + m_\varphi g l_\varphi \Phi \varphi + m_\varphi g l_\varphi \frac{\varphi^2}{2} \qquad (14)$$

$$\Theta = \frac{1}{2} k_\Phi \dot\Phi^2 + \frac{1}{2} k_\varphi \dot\varphi^2 \qquad (15)$$

The physical model of tank and ship represents a two-DOF object, hence the expression (12) is the set of two equations. One of them is the equation of derivatives of relevant energies respective to Φ, (q_1), and another - respective to φ, (q_2). The wave-induced moment $M_{qj} \equiv M_{q1} = M_\lambda$, complying with the equation, is introduced to the right-hand side of the ship roll equation. As the passive tank is considered, $M_{qj} \equiv M_{q2} = 0$ is introduced to the right-hand side of the equation of motions of liquid in the tank. After calculation of energy differentials, replacements and ordering the equations, one obtains :

$$\begin{cases} [m_\Phi l_\Phi^2 + m_\varphi (R_0' - l_\varphi)^2] \ddot\Phi + k_\Phi \dot\Phi + [m_\Phi g l_\Phi - m_\varphi g (R' - l_\varphi)] \Phi + \\ - m_\varphi (R_0' - l_\varphi) l_\varphi \ddot\varphi + m_\varphi g l_\varphi \varphi + \\ - m_\varphi (R_0' - l_\varphi) \dot V_\lambda = D h \alpha_m \sin\omega_\lambda t \\ m_\varphi l_\varphi^2 \ddot\varphi + k_\varphi \dot\varphi + m_\varphi g l_\varphi \varphi - m_\varphi (R_0' - l_\varphi) l_\varphi \ddot\Phi + \\ + m_\varphi g l_\varphi \Phi + m_\varphi l_\varphi \dot V_\lambda = 0 \end{cases} \qquad (16)$$

The final form of the equations is obtained by introducing the value $\dot V_\lambda = g\alpha_m \sin\omega_\lambda t$ into (16), with making use of (10).

Therefore :

$$\begin{cases} [m_\Phi l_\Phi^2 + m_\varphi (R_0' - l_\varphi)^2] \ddot\Phi + k_\Phi \dot\Phi + \\ + [m_\Phi g l_\Phi - m_\varphi g (R' - l_\varphi)] \Phi - m_\varphi (R_0' - l_\varphi) l_\varphi \ddot\varphi + \\ + m_\varphi g l_\varphi \varphi = [D h + m_\varphi g (R_0' - l_\varphi)] \alpha_m \sin\omega_\lambda t \\ m_\varphi l_\varphi^2 \ddot\varphi + k_\varphi \dot\varphi + m_\varphi g l_\varphi \varphi - m_\varphi (R_0' - l_\varphi) l_\varphi \ddot\Phi + \\ + m_\varphi g l_\varphi \Phi = - m_\varphi l_\varphi g \alpha_m \sin\omega_\lambda t \end{cases} \qquad (17)$$

Under the made assumptions the expression (17) represents the full form of the roll motion equations of the ship fitted with passive stabilizing tank. The only simplification represents not taking into account the mutual influence of ship's oscillations and motions of liquid in tank, due to damping. If an influence of sway is neglected the motion equations take the following form :

$$\begin{cases} [m_\Phi l_\Phi^2 + m_\varphi (R_0' - l_\varphi)^2] \ddot\Phi + k_\Phi \dot\Phi + \\ + [m_\Phi g l_\Phi - m_\varphi g (R' - l_\varphi)] \Phi - m_\varphi (R_0' - l_\varphi) l_\varphi \ddot\varphi + \\ + m_\varphi g l_\varphi \varphi = D h \alpha_m \sin\omega_\lambda t \\ m_\varphi l_\varphi^2 \ddot\varphi + k_\varphi \dot\varphi + m_\varphi g l_\varphi \varphi + \\ - m_\varphi (R_0' - l_\varphi) l_\varphi \ddot\Phi + m_\varphi g l_\varphi \Phi = 0 \end{cases} \qquad (18)$$

Each of the coefficients appearing in the motion equations should be clearly defined by ship and tank parameters. The equations (17) and (18) derived on the basis of the proposed physical model, satisfy the postulate. Their normalized form is as follows :

$$\begin{cases} \ddot\Phi + \mu_{\Phi 1} \dot\Phi + \omega_{\Phi 1}^2 \Phi - \beta_1 \ddot\varphi + \\ + \gamma_1 \omega_\Phi^2 \varphi = \omega_{D1}^2 \alpha_m \sin\omega_\lambda t \\ \ddot\varphi + \mu_\varphi \dot\varphi + \omega_\varphi^2 \varphi - b \ddot\Phi + \omega_\varphi^2 \Phi = \\ = - \omega_\varphi^2 \alpha_m \sin\omega_\lambda t \end{cases} \quad \text{– accounting for sway} \qquad (19)$$

$$\begin{cases} \ddot\Phi + \mu_{\Phi 1} \dot\Phi + \omega_{\Phi 1}^2 \Phi - \beta_1 \ddot\varphi + \\ + \gamma_1 \omega_\Phi^2 \varphi = \omega_{D2}^2 \alpha_m \sin\omega_\lambda t \\ \ddot\varphi + \mu_\varphi \dot\varphi + \omega_\varphi^2 \varphi - b \ddot\Phi + \\ + \omega_\varphi^2 \Phi = 0 \end{cases} \quad \text{– not accounting for sway} \qquad (20)$$

Next, the expression appearing at $\ddot\Phi$ in the equations (18) was marked K and the following notation was applied :

$$[m_\Phi l_\Phi^2 + m_\varphi (R_0' - l_\varphi)^2] = K \quad , \quad k_\Phi / K = \mu_{\Phi 1}$$

$$[m_\Phi g l_\Phi - m_\varphi g (R' - l_\varphi)] / K = \omega_{\Phi 1}^2$$

$$m_\varphi (R_0' - l_\varphi) l_\varphi / K = \beta_1 \quad , \quad m_\varphi g l_\varphi / K = \gamma_1 \omega_\Phi^2$$

$$[D h + m_\varphi g (R_0' - l_\varphi)] / K = \omega_{D1}^2 \quad , \quad D h / K = \omega_{D2}^2$$

$$k_\varphi / m_\varphi l_\varphi^2 = \mu_\varphi \quad , \quad m_\varphi g l_\varphi / m_\varphi l_\varphi^2 = g / l_\varphi = \omega_\varphi^2$$

$$m_\varphi (R_0' - l_\varphi) l_\varphi / m_\varphi l_\varphi^2 = (R_0' - l_\varphi)/l_\varphi = b \qquad (21)$$

The usually available initial data deal separately with ship itself and tank itself. Then $m_\Phi l_\Phi^2$ is given istead of K. Hence the coefficients in the equations (19) should be expressed by means of :

$$\mu_{\Phi 1} = \frac{\mu_\Phi}{K'} \quad , \quad \omega_{\Phi 1}^2 = \frac{\omega_\Phi^2}{K''} \quad , \quad \beta_1 = \frac{\beta}{K'}$$

$$\gamma_1 = \frac{\gamma}{K'} \quad , \quad \omega_{D1}^2 = \frac{\omega_\Phi^2}{K'} \quad , \quad \omega_{D2}^2 = \omega_\Phi^2 \frac{K'''}{K'} \qquad (22)$$

where :

$$\mu_\Phi = k_\Phi / m_\Phi l_\Phi^2 \quad , \quad \omega_\Phi^2 = m_\Phi g l_\Phi / m_\Phi l_\Phi^2 = g / l_\Phi^2$$

The quantities μ_Φ, ω_Φ^2, β, γ, K', K'', K''', expressed respectively by the physical model parameters and the ship and tank parameters, have the same form as that given in the expressions (23), which makes it possible to examine structure of each of the coefficients appearing in the motion equations.

PHYSICAL MODEL	SHIP
$\mu_\Phi = \frac{k_\Phi}{m_\Phi l_\Phi^2}$	$\mu_\Phi = \frac{k_\Phi}{I_x + i_{xx}} = \frac{k_\Phi}{m_s r_{sm}^2}$
$\omega_\Phi^2 = \frac{m_\Phi g l_\Phi}{m_\Phi l_\Phi^2} = \frac{g}{l_\Phi}$	$\omega_\Phi^2 = \frac{m_s g h_s}{I_x + i_{xx}} = \frac{g h_s}{r_{sm}^2}$
$\gamma = \frac{m_\varphi l_\varphi}{m_\Phi l_\Phi}$	$\gamma = \xi \frac{r_z}{h_s}$ (23)
$\beta = \frac{m_\varphi}{m_\Phi} \frac{(R_0' - l_\varphi)}{l_\Phi} \frac{l_\varphi}{l_\Phi}$	$\beta = \gamma \left(\frac{R_0 k_0}{\sqrt{l_\Phi r_z}} - 1 \right) \frac{1}{k_0^2}$
$b = \frac{R_0'}{l_\varphi} - 1$	$b = \frac{R_0 k_0}{\sqrt{l_\Phi r_z}} - 1$

PHYSICAL MODEL	SHIP
$K' = 1 + \dfrac{m_\varphi}{m_\Phi} \dfrac{(R'_0 - l_\varphi)^2}{l_\Phi^2}$	$K' = 1 + \beta^2 \dfrac{k_0^2}{\gamma}$
$K'' = \dfrac{K'}{1 - \dfrac{m_\varphi}{m_\Phi} \dfrac{(R' - l_\varphi)}{l_\Phi}}$	$K'' = \dfrac{K'}{1 - \gamma \left(\dfrac{R}{r_z} - 1 \right)}$
$K''' = \dfrac{m_s + m_z}{m_\Phi} \dfrac{h}{l_\Phi}$	$K''' = (1 + \xi) \dfrac{h}{h_s}$ (23)
$R' = R \dfrac{l_\varphi}{r_z}$	$R = z_p + r_z - T - \Delta T/2 - \rho_c$
$R'_0 = R_0 \sqrt{\dfrac{l_\varphi}{r_z}}$	$R_0 = z_p + r_z - T - \Delta T + a_w$

$$h = h_s + \frac{\xi}{1+\xi} \left(T + \frac{\Delta T}{2} + \rho_c - z_p - h_s \right)$$

$$\rho_c = r_s \, (1 - \delta/\alpha) \frac{2}{2 + (\delta/\alpha)\xi} \qquad a_w = 0.43 \, [(T + \Delta T) - z_G] + 0.1B$$

$$\xi = m_z/m_s \;,\quad k_0 = \omega_\varphi/\omega_\Phi \;,\quad \omega_0 = \omega_\lambda/\omega_\Phi \;,\quad \mu_{\varphi 0} = \mu_\varphi/\omega_\Phi$$

for : $\Delta T/2 \approx 0$ and $\rho_c \approx 0$ it yields :

$$R \approx z_p + r_z - T \qquad R_0 \approx R + a_w$$

$$h \approx h_s + \frac{\xi}{1+\xi} (T - z_p - h_s)$$

Solutions of the motion equations, transfer functions

The general form of ship roll transfer functions obtained as a result of solving the equations (19) and (20), is the following :

$$\frac{\Phi}{\alpha_m} = K_S \sqrt{\frac{A^2 + B^2}{C^2 + D^2}} \qquad (24)$$

For (19) where sway is accounted for, one obtains :

$$A = \mu_{\varphi 0} \, k_0 \, \omega_0$$

$$B = k_0^2 (1 + \gamma) - \omega_0^2 (1 - \beta \, k_0^2)$$

$$C = -K' \, (1/K'' - \omega_0^2)(k_0^2 - \omega_0^2) + (\gamma + \beta\omega_0^2)(k_0^2 + b \, \omega_0^2) + \mu_{\Phi 0} \, \mu_{\varphi 0} \, k_0 \, \omega_0^2$$

$$D = K' \, \mu_{\varphi 0} \, k_0 \, \omega_0 \, (1/K'' - \omega_0^2) + \mu_{\Phi 0} \, \omega_0 \, (k_0^2 - \omega_0^2)$$

$$K_s = 1$$

For (20), without accounting for sway, it is :

$$A = \mu_{\varphi 0} \, k_0 \, \omega_0 \;,\quad B = k_0^2 - \omega_0^2$$

$$C \text{ and } D \text{ as for (20)} \;,\quad K_s = K''' = \frac{m_s + m_z}{m_\Phi} \frac{h}{l_\Phi}$$

The presented expressions which make it possible to calculate transfer functions for the ship fitted with stabilizing tank, directly provide the following options :

- ✦ to follow changes appearing in solutions depending on an applied variant of the equations
- ✦ to determine in which way changes of the main parameters of tank and ship influence the calculated functions.

In the equations based on ship and tank data axcess to variables and parameters is easy and direct. It is possible to directly change such parameters as: z_p - tank position, k_0 - tuning factor of natural frequencies of tank and ship, r_z -metacentric radius of tank , i.e. its type, ξ - i.e. amount of liquid in tank, D_s and h_s - i.e. ship loading state.

The assumed form of the data and auxiliary values, (23), including the variables :

- $\omega_0 = \omega_\lambda/\omega_\Phi$ - dimensionless excitation frequency, an independent variable
- $\mu_{\varphi 0} = \mu_\varphi/\omega_\Phi$ - dimensionless damping coefficient of motions of liquid in tank, the parameter for sucessive realizations of functions of a given type
- $k_0 = \omega_\varphi/\omega_\Phi$ - relative natural frequency of oscillations of liquid in tank, possible to be used as an intermediate parameter
- $\xi = m_z/m_s$ - relative mass of liquid in tank, possible to be used as an intermediate parameter;

all of them make it possible to use a single, simple set of initial data, regardless of simplification level of the equations in question.

APPLICATION OF THE MATHEMATICAL MODEL

The obtained roll transfer functions make it possible to follow influence of the main ship and tank parameters on ship roll amplitudes, and thus to ensure stabilization effectiveness of a given tank. To achieve courses of relevant functions the data of the ship and tank considered in [16] were selected. A correct course of the achieved functions serves also as a check of correction of the formulae based on the presented physical and mathematical models.

The ship with stabilizing tank acc.to [16] :

Ship parameters :

- length b.p. — L_{bp} = 266.00 m
- breadth — B = 42.50 m
- draught — T = 10.85 m
- mass displacement — $\Delta \equiv m_s$ = 97800 t
- height of centre of gravity — z_{Gs} = 15.40 m
- initial metacentric height (without tank) — $GM_s \equiv h_s$ = 3.576 m
- block coefficient — δ = 0.797
- waterplane coefficient — α = 0.831
- natural roll period (natural frequency) — τ_Φ = 17.10 s (ω_Φ = 0.37 1/s)
- roll damping coefficient — μ_Φ = 0.070 1/s.

Tank parameters (see Fig.1) :

- length — l_z = 13.60 m
- breadth — b_z = 40.89 m

- design height $h_k = 4.79$ m
- natural frequency of motions of liquid in tank $\omega_\varphi = 0.36$ 1/s
- level of liquid in tank $h_z = 2.20$ m
- mass of liquid in tank at $\rho = 1.00$ t/m^3 $m_z = 1224$ t
- tank bottom height over the plane of reference $z_d = 20.00$ m
- tuning factor $k_0 = \omega_\varphi/\omega_\Phi = 0.97$
- relative mass of liquid in tank $\xi = m_z/m_s = 0.0125$

The diagrams shown in Fig.8 and 9 illustrate the roll transfer functions of the ship with tank. The dimensionless damping coefficient of motions of liquid in tank, $\mu_{\varphi 0}$, is a parameter for particular curves. Fig.8 deals with the full set of equations in which sway is accounted for, and Fig.9 - the full set but without taking into account those oscillations. The presented diagrams are usually used for choosing an optimum value of the damping coefficient $\mu_{\varphi 0}$, and after that – for evaluating the maximum stabilizing effect [5, 12, 15]. A value of the damping coefficient for tank is so selected as in the middle, "design" range of the diagrams the difference between the roll amplitude of the non-stabilized ship, which corresponds to the value $\mu_{\varphi 0} = \infty$, and a selected value e.g. $\mu_{\varphi 0} = 0.6$; were the greatest. The value of this difference, determined for the maximum amplitude of the non-stabilized ship, hence for the excitation resonant frequency, is the stabilizing effect searched for. A value of $\mu_{\varphi 0}$ should be selected in such a way as in a range far from resonance e.g. outside the characteristic nodes of the transfer functions' diagram, an increment of amplitudes, usually appearing there for stabilized ship, were the smallest. The diagrams also show that for $\mu_{\varphi 0} = 0$, which is possible theoretically only, such excitation frequency exists at which the stabilizing effect amounts to 100%. The stabilizing effect expressed in percent, is the ratio of the roll amplitude reduction and non-stabilized amplitude.

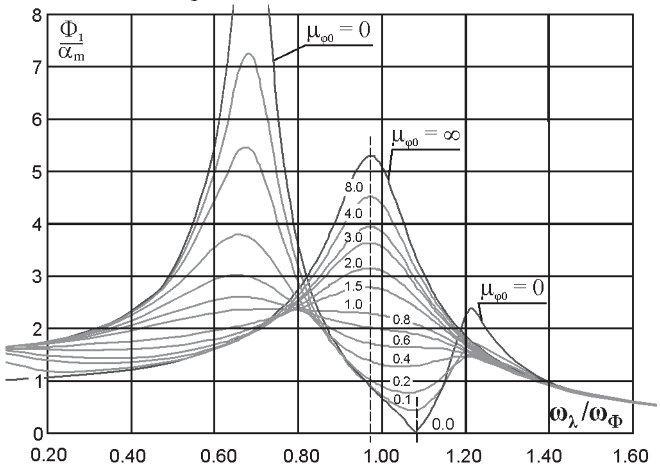


Fig. 8. Diagrams of roll transfer functions with taking into account sway

Fig.8 it can be observed that when sway is accounted for, the zero value of roll amplitude occurs at a frequency other than resonant one of non-stabilized ship. It means that the maximum amount of roll energy is absorbed by the tank from the ship at that other frequency. Hence in the case if sway is accounted for no phenomenon of double resonance can be directly observed.

Influence of changing the tank parameters on course of roll transfer function

It was assumed that in the preliminary design stage it is more convenient to make use of the full-form equations but without taking into account sway. Information this way obtained is more transparent and more useful for making relevant estimations and comparisons.

In Fig.10 based on Fig.9 , only the courses for the dimensionless damping coefficients of liquid in tank, $\mu_{\varphi 0} = \infty$ and $\mu_{\varphi 0} = 0.8$, are left. Fig.10 is further used asa a basis for making comparisons of tank stabilizing effectiveness depending on changes introduced to its location or other parameters. Fig.11 shows in which way the roll reduction effect is influenced by a change in vertical location of the stabilizing tank. Next, in Fig.12 it is presented to which extent a change of tank metacentric radius impairs ship roll amplitudes. From the presented diagrams it can be easily observed that their courses are almost in the same way unfavourably influenced by the decreasing of

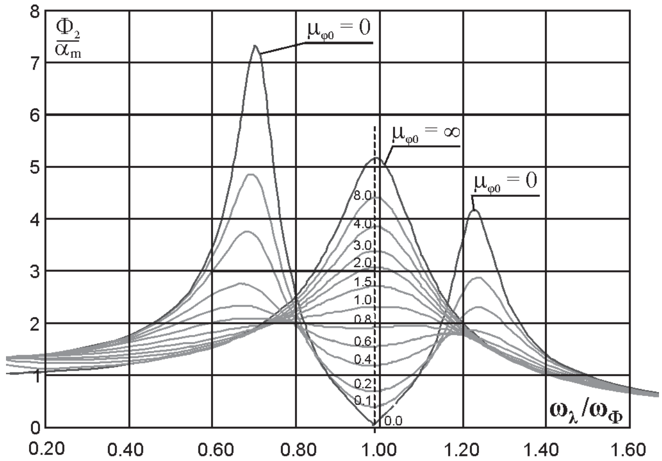


***Fig.9.** Diagrams of roll transfer functions without taking into account sway*

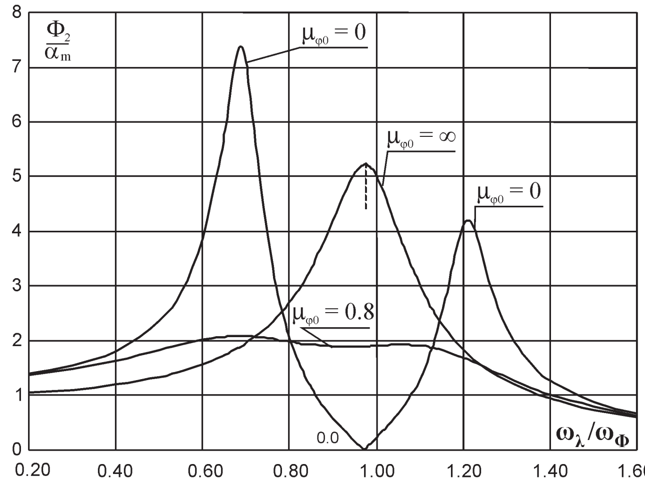


***Fig. 10.** Main transfer functions, without accounting for sway*
$k_0 = 0.97$; $\xi = 0.013$; $z_p = 20.00$ m ; $r_z = 63.33$ m

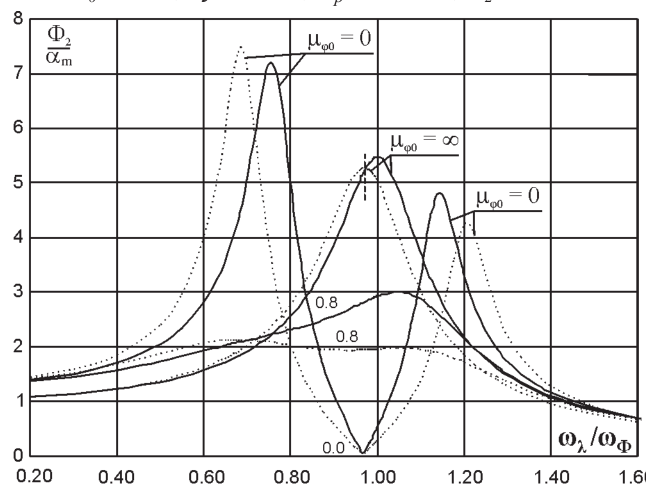


***Fig. 11.** Influence of the vertical coordinate of tank location,*
——— $z_p = 2.00$ m ; ---------- $z_p = 20.00$ m (Fig. 10)

tank elevation and the decreasing of tank metacentric radius. In both the cases the stabilizing effect is lower because the roll amplitudes of the stabilized ship are greater. By making use of the physical model (Fig.5, Fig.6, Fig.7) it can be observed that in both cases this is caused by changing the distance between the tank pendulum suspension point and the pivoting point of the whole system of pendulae of the physical model.

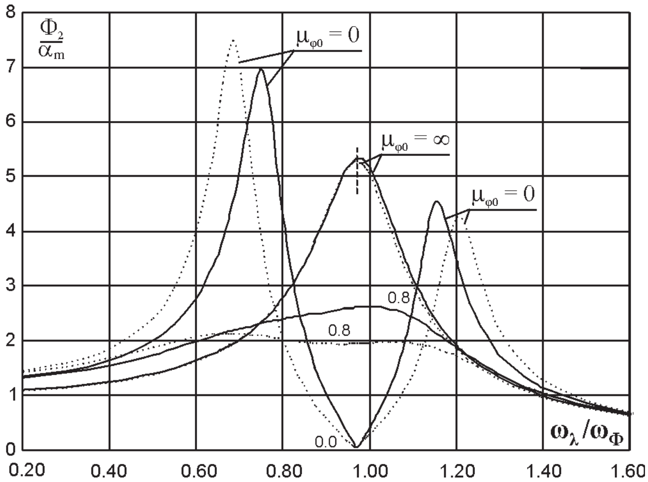


Fig. 12. *Influence of the tank metacentric radius - change of type of tank* ——— $r_z = 46.73$ *m* ; ---------- $r_z = 63.33$ *m (Fig. 10)*

⇨ Generally, it can be stated that the higher located the passive stabilizing tank metacentre over the ship rolling axis, the greater the stabilizing effects.This is an important guideline for designers.

⇨ Another important conclusion is : that type of selected passive stabilizing tank is better whose metacentric radius is the greatest, at maintaining the remaining parameters, such as working liquid mass and its natural period of motion in tank, constant.

⇨ It is proposed to assume the ratio of the length of the tank metacentric radius and that of the tank mathematical pendulum as a relative measure of tank stabilizing quality, independent on tank dimensions and mass of its working liquid, and expressed as follows :

$$C_Z = r_Z/l_\varphi \qquad (25)$$

For tanks of perpendicular sides the quantity C_Z is practically constant, independent on their dimensions and the working liquid level h_Z . For instance for the cubicoid tank and "Flume" tank it amounts to: $C_{Z1} \cong 0.82$ and $C_{Z2} \cong 0.67$, respectively (the C_{Z2} value deals with the "Flume" tank having the dimensions of the middle throat equal to a half of tank length and a half of tank breadth, respectively). C_Z values start to change distinctly for greater values of the liquid level in tank, h_Z , (i.e. for $h_Z > 0.2b_Z$) which is associated with the necessity of using more complex relationships in determining the natural periods of working liquid motions. Basing on the above given statements one can postulate the following thesis :

Effectiveness of the passive stabilizing tanks of free liquid surface increases when the ratio of their metacentric radius and length of mathematical pendulum increases;
the pendulum length is determined by the natural period of working liquid in tank. The ratio in question can be called
"**the stabilizing effectiveness of tank of a given type**".

The presented relationships open an interesting area for searching for optimum solutions of passive stabilizing tanks having free working liquid surface,
and also for further research on ship roll stabilization.

RECAPITULATION AND CONCLUSIONS

❍ The presented mathematical model can be deemed well adjusted for designing the passive stabilizing tank to be installed in ship. The elaborated calculation method can be used as a tool for aiding preliminary design stage of a ship fitted with passive stabilizing tank.

❍ For correct design of stabilizing tanks the following three principles formulated on the basis of the presented model of ship with stabilizing tank, are important :

- ❖ the higher located metacentre of passive stabilizing tank over ship rolling axis, the greater expected stabilizing effects
- ❖ that type of passive stabilizing tank is better whose metacentric radius is the greatest, at maintaining the remaining parameters such as working liquid mass and natural period of motions of liquid, constant
- ❖ the higher the ratio of the tank metacentric radius and the mathematical pendulum length determined by natural period of motions of liquid in tank, the higher the stabilizing effectiveness of a selected type of passive stabilizing tank with free liquid surface.

Moreover, it should be added that :

- ➢ in spite of the simplifications resulting from the assumed physical model, the achieved solutions provide sufficiently exact description of roll of a ship fitted with stabilizing tank. The wide range of the simplifications was introduced in order to reach "design usufulness" of the elaborated equations and their solutions
- ➢ the way of accounting for sway in the obtained equations arouses some reservations with regard to the introduced important simplifications. Nonetheless it was proved that in the case of real sea conditions and accounting for sway the so called double resonance did not directly appear for a ship with stabilizing tank. For this reason its stabilizing effect may appear lower than that expected.

NOMENCLATURE

a_W	- distance between rolling axis and waterplane, [m]
D, D_s	- buoyance force of ship with and without tank, respectively, [kN]
h, h_s	- initial metacentric height, not corrected regarding free-surface influence, for ship with and without tank, respectively, [m]
k_Φ, μ_Φ and $\mu_{\Phi 0}$	- ship roll damping coefficient, kg m²/s, 1/s, and [-]
k_φ, μ_φ and $\mu_{\varphi 0}$	- damping coefficient of motions of liquid in tank, [kg m²/s, 1/s] and [-]
$k_0 = \omega_\varphi/\omega_\Phi$	- relative natural frequency of motions of liquid in tank, tuning factor, [-]
l_Φ	- length of ship mathematical pendulum, [m]
l_φ	- length of tank mathematical pendulum, [m]
M_λ	- ship roll excitation moment, [Nm]
m_s	- ship mass, [t]
m_z	- mass of liquid in tank, [t]
m_Φ	- mass of ship mathematical pendulum, [t]
m_φ	- mass of tank mathematical pendulum, [t]
R	- distance between metacentre of tank and metacentre of increment of ship immersed volume, corresponding to mass of liquid in tank, [m]
R'	- distance in physical model corresponding to R of ship, [m]
R_0	- distance between tank metacentre and ship rolling axis, [m]
R'_0	- distance between tank pendulum suspension point and physical model pivoting axis, [m]
r_s	- metacentric radius of ship without tank, [m]

r_{sm} - ship mass inertia radius, [m]
r_z - tank metacentric radius, [m]
r_{zm} - mass inertia radius of liquid in tank, [m]
t - time, [s]
T - draft, [m]
V_λ - horizontal component of velocity of mass of liquid in tank, due to ship sway, [m/s]
y_λ - ship sway, [m]
z_G - height of mass centre of ship with tank, over reference plane, [m]
z_{Gs} - height of mass centre of ship without tank, over reference plane, [m]
z_p - height of mass centre of liquid in tank, over reference plane, [m]
α - waterplane coefficient, [-]
α_λ, α_m - amplitude and effective amplitude of wave slope angle, respectively, [rad, °]
δ - ship block coefficent, [-]
κ_B, κ_T - correction coefficients of wave slope angle, dependent on B/λ and T/λ, where : λ - length of ship roll inducing wave
$\xi = m_z/m_s$ - relative mass of liquid in tank, [-]
ρ_c - metacentric radius of displacement layer of ΔT thickness, [m]
τ_Φ - natural oscillation period of ship pendulum and real ship, [s]
τ_φ - natural oscillation period of tank pendulum and of liquid's motions in tank, [s]
Φ - ship heeling angle, [rad, °]
φ - angular displacement of mass centre of liquid in tank, respective to ship, (mean slope angle of liquid surface in tank, respective to ship), [rad, °]
$\omega_0 = \omega_\lambda/\omega_\Phi$ - relative excitation frequency, [-]
ω_λ - wave frequency, [1/s]
ω_Φ - ship roll natural frequency, [1/s]
ω_φ - natural frequency of liquid's motions in tank, [1/s]

BIBLIOGRAPHY

1. Balcer L.: *Location of ship rolling axis*. Polish Maritime Research, Vol. 11, No. 1(39). Gdańsk, 2004
2. Bass D.W.: *Roll Stabilization for Small Fishing Vessels Using Paravanes and Anti-Roll Tanks*. Marine Technology, Vol. 35, No.2, April 1998
3. Borodaj I.K., Niecvietajev J. A.: *Kačka sudov na morskom volnienii*. Sudostrojenije. Leningrad, 1969
4. Bruce L. Hutchison: *The Transverse Plane Motions of Ships*. Marine Technology, Vol.28, No.2, March 1991
5. Cholodin A.N., Šmyriev A.N.: *Moriechodnosť i stabilizacja sudov na volnieni*. Sudostrojenie. Leningrad, 1976
6. Dudziak J.: *Theory of ships* (in Polish). Wydawnictwo Morskie. Gdańsk, 1988
7. Frąckowiak M., Pawłowski M.: *Excersises in ship hydromechanics* (in Polish). Technical University of Gdańsk. Gdańsk, 1978
8. Goodrich G. J.: *The Development and Design of Passive Roll Stabilisers*, Shipping World and Shipbuilder, July 1968
9. Lejko J.: *Dynamics of material systems* (in Polish). Part I and II. Państwowe Wydawnictwo Naukowe. Warszawa, 1959/1961
10. Shen Hua, Huang Dingliang: *Roll Motion and Stabilising of Box-Shaped Ship with Extra Shallow Draft*. International Shipbuilding Progress, Vol. 45 No.443 (1998)
11. Siemionow-Tian-Szanskij W. W., Blagowieszczenskij S. N., Holodin A. N.: *Kačka korablia*. Sudostrojenie. Leningrad, 1969
12. Šmyriev A. N., Morenšyldt W. A., Ilina S. G., Goldin A. I.: *Uspokoitieli kački sudov*, Sudostrojenije, Leningrad 1972
13. Vasta J., Giddings A. J., Taplin A., Stilwell J. J.: *Roll Stabilization by Means of Passive Tanks*. Trans. SNAME, Vol. 69 (1961)
14. Wilson P. A.: *Lateral Force Estimator and Its Uses In Controlling and Stabilising Ships*. Ship Manoeuvring Workshop : Manoeuvring Qualities in Ship Design. Technical University of Gdańsk and Foundation for Safety of Navigation and Environment Protection. Iława-Poland, 4 - 7 October, 1993
15. Zdybek T.: *Passive stabilizing tanks* (in Polish). Wydawnictwo Morskie. Gdańsk, 1982
16. Zdybek T.: *The Use of Bench Test Results for Calculating Roll Response of the Tank Stabilised Ship*. International Shipbuilding Progress, Vol. 27, No 308, April 1980

CONTACT WITH THE AUTHOR

Ludwik Balcer, D.Sc.
Faculty of Ocean Engineering
and Ship Technology,
Gdańsk University of Technology
80-952 Gdańsk, POLAND
Narutowicza 11/12
e-mail : balcer@pg.gda.pl

Conference

Scientific Seminar of Regional Group

of the Section on Exploitation Foundations

On 24 June 2004 Gdynia Maritime University (GMU) hosted the 2nd – in-this-year scientific seminar of the Regional Group of the Section on Exploitation Foundations, Machine Building Committee, Polish Academy of Sciences.

Four papers whose authors came from the scientific staff of GMU Mechanical Faculty, were presented during the seminar :

- ★ *Results of service investigations of transverse sliding bearings lubricated with the use of non-Newtonian oils* by A. Miszczak, D.Sc.
- ★ *Problems associated with modeling NOx emission from ship two-stroke engine* – by J. Kowalski, M.Sc.
- ★ *Influence of lubricating oil contamination on wear of elements working in mixed friction conditions* by A. Młynarczyk, M.Sc., Eng.
- ★ *Selected problems associated with diagnostics of one-stage refrigerating system* – by T. Hajduk, M.Sc.

The seminar was ended by presentation of the laboratory facilities of GMU Mechanical Faculty to the seminar participants.

Photo: C. Spigarski

Fatigue "safe-life" criterion for metal elements under multiaxial static and dynamic random loads

Janusz Kolenda
Gdańsk University of Technology
Naval University of Gdynia

ABSTRACT

Random stress with Cartesian components of known statistical moments of their mean values and power spectral densities of stochastic stress processes is considered. To account for the mean stress effect the generalised Soderberg criterion for ductile materials is employed. An equivalent stress with periodic (in the mean-square sense) components is defined by means of the equivalence conditions based on the average strain distortion energy. Also, is formulated the fatigue "safe-life" design criterion which covers the conditions of both static strength and fatigue safety and includes yield strengths and fatigue limits which : have simple physical interpretation, can be determined by uniaxial tests, are directly related to the applied loads, and can reflect material anisotropy.

Key words : design criteria, multiaxial loading, random stress, mean stress effect

INTRODUCTION

The past decade has shown that fatigue is still a great challenge for the engineering community. The reasons of it , among other, may be the random or stochastic character of most loads that occur in nature. Therefore in this paper being direct continuation of the author's paper [1] devoted to fatigue safety of metal elements under deterministic loads, an attempt is made to formulate the fatigue "safe-life" design criterion for metal elements by using probabilistic approach. It is assumed that the considered stress is described by its Cartesian components as follows :

$$\sigma_i(t) = c_i + s_i(t) \quad , \quad i = x,\ y,\ z,\ xy,\ yz,\ zx \tag{1}$$

where :

- c_i - random mean values of known 2nd statistical moments
- $s_i(t)$ - zero mean stochastic processes of known power spectral densities, which are stationary (in the wide sense), stationary correlated with each other and statistically independent of the mean values c_i .

For the sake of brevity the stress components $\sigma_z(t)$, $\sigma_{yz}(t)$ and $\sigma_{zx}(t)$ are dropped.

In view of practical calculations, the actual stress components should be modelled with equivalent stress components of a relatively simpler form by means of an appropriate fatigue strength theory. For the multiaxial stress when axial forces, bending moments and torsional loads vary in time none of the fatigue strength theories is universally accepted and all fatigue criteria usually demonstrate large scatter [2]. On the other hand, in certain dynamic cases the conventional strength theories are considered to be satisfactory [3÷5]. From some fatigue tests it has been concluded that also the criterion based on the average strain energy appears promising [6]. However, bearing in mind that for ductile metals the Huber-von Mises-Hencky strength theory is commonly accepted [7, 8], the average strain distortion energy is here, like in [1] , employed.

The presented paper is aimed at the design criterion which would cover both the static strength conditions and fatigue safety requirements under multiaxial stress. For this purpose use can be made of the generalised Soderberg criterion for non-zero mean in-phase stress [1] :

$$\left[\sum_i \left(\frac{c_i}{R_i}\right)^2 - \frac{c_x c_y}{R_x R_y}\right]^{1/2} + \left[\sum_i \left(\frac{a_i}{F_i}\right)^2 - \frac{a_x a_y}{F_x F_y}\right]^{1/2} \le 1 \quad , \quad i = x,\ y,\ xy \tag{2}$$

where :

- R_i - yield strength relevant to the mean value of i-th stress component, c_i
- F_i - fatigue limit under fully reversed load relevant to the amplitude of i-th stress component, a_i .

As far as the computational effectiveness is concerned, spectral criteria can be very advantageous [9]. Therefore the applied equivalence conditions are transformed into the frequency domain.

EQUIVALENT STRESS UNDER MULTIAXIAL STATIC AND DYNAMIC LOADS

The equation (2) suggests to model the stress components (1) by the equivalent stress components :

$$\sigma_i^{(eq)}(t)=c_i^{(eq)}+s_i^{(eq)}(t) \quad , \quad i=x,\ y,\ xy \tag{3}$$

where :

$c_i^{(eq)}$ - random mean value of i-th equivalent stress component

$s_i^{(eq)}(t)$ - i-th zero mean stochastic process.

It is assumed that the processes $s_i^{(eq)}(t)$ are periodically stationary (in the mean-square sense [10]), stationary correlated with each other and statistically independent of the mean values $c_i^{(eq)}$, which are sought in the form :

$$s_i^{(eq)}(t)=a_i^{(eq)}\sin\left(\omega_{eq}t+\varphi_i\right)= \\ =a_{i1}\exp\left(j\omega_{eq}t\right)+a_{i2}\exp\left(-j\omega_{eq}t\right)\ ,\ i=x,\ y,\ xy \tag{4}$$

where :

$a_i^{(eq)},\varphi_i$ - random amplitude and phase angle of i-th equivalent stress component.

$$a_{i1}=\frac{a_i^{(eq)}}{2j}\exp(j\varphi)\ ,\quad a_{i2}=a_{i1}^* \tag{5}$$

$$\langle a_{i1}\rangle=\langle a_{i2}\rangle=\langle a_{i1}^*a_{i2}\rangle=\langle a_{i2}^*a_{i1}\rangle=0 \tag{6}$$

$$\langle a_{x1}^*a_{y2}\rangle=\langle a_{x2}^*a_{y1}\rangle=0 \tag{7}$$

ω_{eq} - equivalent circular frequency

$(\bullet)^*$ - complex conjugate

$\langle\bullet\rangle$ - expected value

In order to calculate the amplitudes of the equivalent stress components the following equivalence condition [1] is used at the beginning :

$$\frac{1}{T}\int_0^T\phi_{eq}(t)dt=\frac{1}{T}\int_0^T\phi(t)dt \tag{8}$$

where : T is the averaging time

$$\phi_{eq}(t)=\frac{1+\nu}{3E}\left[\begin{array}{c}\left(\sigma_x^{(eq)}\right)^2+\left(\sigma_y^{(eq)}\right)^2+ \\ -\sigma_x^{(eq)}\sigma_y^{(eq)}+3\left(\sigma_{xy}^{(eq)}\right)^2\end{array}\right] \tag{9}$$

is the strain energy of distortion per unit volume in the equivalent stress state and

$$\phi(t)=\frac{1+\nu}{3E}\left(\sigma_x^2+\sigma_y^2-\sigma_x\sigma_y+3\sigma_{xy}^2\right) \tag{10}$$

is that in the actual stress state.

By applying (8) through (10), the following equivalence conditions can be written :

$$\frac{1}{T}\int_0^T\left[\sigma_i^{(eq)}(t)\right]^2dt=\frac{1}{T}\int_0^T\sigma_i^2(t)dt\ \ ,\ i=x,y,xy \tag{11}$$

$$\frac{1}{T}\int_0^T\sigma_x^{(eq)}(t)\sigma_y^{(eq)}(t)dt=\frac{1}{T}\int_0^T\sigma_x(t)\sigma_y(t)dt \tag{12}$$

Such equations of integral time averages are not convenient for evaluation of parameters of the equivalent stress components by using spectral data therefore they were replaced by the equations of ensemble averages :

$$\left\langle\left[\sigma_i^{(eq)}(t)\right]^2\right\rangle=\left\langle\sigma_i^2(t)\right\rangle\ ,\ i=x,y,xy \tag{13}$$

$$\left\langle\sigma_x^{(eq)}(t)\sigma_y^{(eq)}(t)\right\rangle=\left\langle\sigma_x(t)\sigma_y(t)\right\rangle \tag{14}$$

To obtain the frequency domain formulation of (13) they were rewritten in terms of correlation functions as follows :

$$\left\langle\begin{array}{l}\left[c_i^{(eq)}+a_{i1}^*\exp\left(-j\omega_{eq}t_1\right)+a_{i2}^*\exp\left(j\omega_{eq}t_1\right)\right]\cdot \\ \cdot\left[c_i^{(eq)}+a_{i1}\exp\left(j\omega_{eq}t_2\right)+a_{i2}\exp\left(-j\omega_{eq}t_2\right)\right]\end{array}\right\rangle= \\ =\left\langle\left[c_i+s_i^*(t_1)\right]\left[c_i+s_i(t_2)\right]\right\rangle \tag{15}$$

Stationarity, in the wide sense, of the processes $s_i(t)$ implies that :

$$\left\langle\left[c_i+s_i^*(t_1)\right]\left[c_i+s_i(t_2)\right]\right\rangle=\left\langle c_i^2\right\rangle+K_i(\tau) \tag{16}$$

where :

$\tau = t_2 - t_1$

and :

$$K_i(\tau)=\left\langle s_i^*(t_1)s_i(t_2)\right\rangle$$

is the autocorrelation function of the process $s_i(t)$. In accordance with (3) through (6), and (16), one gets from (15) :

$$\left\langle\left(c_i^{(eq)}\right)^2\right\rangle+\frac{1}{4}\left\langle\left(a_i^{(eq)}\right)^2\right\rangle\left[\exp\left(j\omega_{eq}\tau\right)+ \\ +\exp\left(-j\omega_{eq}\tau\right)\right]=\left\langle c_i^2\right\rangle+K_i(\tau) \tag{17}$$

Thus :

$$\left\langle\left(c_i^{(eq)}\right)^2\right\rangle=\left\langle c_i^2\right\rangle \tag{18}$$

$$\frac{1}{4}\left\langle\left(a_i^{(eq)}\right)^2\right\rangle\left[\exp\left(j\omega_{eq}\tau\right)+\exp\left(-j\omega_{eq}\tau\right)\right]=K_i(\tau) \tag{19}$$

Fourier transformation of (19) gives :

$$\frac{1}{4}\left\langle\left(a_i^{(eq)}\right)^2\right\rangle\left[\delta\left(\omega-\omega_{eq}\right)+\delta\left(\omega+\omega_{eq}\right)\right]=S_i(\omega) \tag{20}$$

where :

δ - Dirac's delta function

$S_i(\omega)$ - power spectral density of the process $s_i(t)$.

The mean-square value of the amplitude of i-th equivalent stress component can be estimated with the use of (20) by its integration over the whole frequency range, which yields :

$$\frac{1}{2}\left\langle\left(a_i^{(eq)}\right)^2\right\rangle=\int_{-\infty}^{\infty}S_i(\omega)d\omega \tag{21}$$

and :

$$\left\langle\left(a_i^{(eq)}\right)^2\right\rangle=2\int_{-\infty}^{\infty}S_i(\omega)d\omega \tag{22}$$

Similarly, stationary cross - correlation of the processes $\sigma_x(t)$ and $\sigma_y(t)$ requires that :

$$\left\langle \sigma_x^*(t_1)\sigma_y(t_2)\right\rangle = \left\langle c_x c_y\right\rangle + K_{x,y}(\tau) \tag{23}$$

where :

$$K_{x,y}(\tau) = \left\langle s_x^*(t_1)s_y(t_2)\right\rangle$$

is the cross-correlation function of the processes $s_x(t)$ and $s_y(t)$.

By proceeding with (14) – in the same way as above – the following was obtained :

$$\begin{aligned}&\left\langle c_x^{(eq)}c_y^{(eq)}\right\rangle + \frac{1}{4}\left\langle a_x^{(eq)}a_y^{(eq)}\cdot\right.\\&\cdot\left\{\exp\left[j\left(\varphi_y-\varphi_x\right)\right]\exp\left(j\omega_{eq}\tau\right)+\right.\\&\left.\left.+\exp\left[-j\left(\varphi_y-\varphi_x\right)\right]\exp\left(-j\omega_{eq}\tau\right)\right\}\right\rangle =\\&=\left\langle c_x c_y\right\rangle + K_{x,y}(\tau)\end{aligned} \tag{24}$$

So :

$$\left\langle c_x^{(eq)}c_y^{(eq)}\right\rangle = \left\langle c_x c_y\right\rangle \tag{25}$$

$$\frac{1}{4}\left\langle a_x^{(eq)}a_y^{(eq)}\left\{\begin{array}{c}\exp\left[j\left(\varphi_y-\varphi_x\right)\right]\cdot\\ \cdot\exp\left(j\omega_{eq}\tau\right)+\\ +\exp\left[-j\left(\varphi_y-\varphi_x\right)\right]\cdot\\ \cdot\exp\left(-j\omega_{eq}\tau\right)\end{array}\right\}\right\rangle = K_{x,y}(\tau) \tag{26}$$

After Fourier transformation of (26) one obtains :

$$\frac{1}{4}\left\langle a_x^{(eq)}a_y^{(eq)}\left\{\begin{array}{c}\exp\left[j\left(\varphi_y-\varphi_x\right)\right]\cdot\\ \cdot\delta\left(\omega-\omega_{eq}\right)+\\ +\exp\left[-j\left(\varphi_y-\varphi_x\right)\right]\cdot\\ \cdot\delta\left(\omega+\omega_{eq}\right)\end{array}\right\}\right\rangle = S_{x,y}(\omega) \tag{27}$$

where : $S_{x,y}(\omega)$ is the cross power spectral density of the processes $\sigma_x(t)$ and $\sigma_y(t)$. Integration of (27) over the whole frequency range yields :

$$\frac{1}{2}\left\langle a_x^{(eq)}a_y^{(eq)}\cos\left(\varphi_y-\varphi_x\right)\right\rangle = \int_{-\infty}^{\infty}S_{x,y}(\omega)d\omega \tag{28}$$

i.e.:

$$\left\langle a_x^{(eq)}a_y^{(eq)}\cos\left(\varphi_y-\varphi_x\right)\right\rangle = 2\int_{-\infty}^{\infty}S_{x,y}(\omega)d\omega \tag{29}$$

FATIGUE "SAFE-LIFE" CRITERION

With regard to (2) through (4), the criterion in question can be formulated as follows :

$$\left\langle f_s^{-1}\right\rangle + \left\langle f_d^{-1}\right\rangle < 1 \tag{30}$$

where :

$$f_s^{-1} = \left[\sum_i\left(\frac{c_i^{(eq)}}{R_i}\right)^2 - \frac{c_x^{(eq)}c_y^{(eq)}}{R_x R_y}\right]^{1/2}, \quad i = x, y, xy \tag{31}$$

$$f_d^{-1} = \left[\sum_i\left(\frac{a_i^{(eq)}}{F_i}\right)^2 - \frac{a_x^{(eq)}a_y^{(eq)}\cos\left(\varphi_y-\varphi_x\right)}{F_x F_y}\right]^{1/2} \tag{32}$$

After expanding the functions f_s^{-1} and f_d^{-1} into Taylor series around expected values of their arguments and retaining the linear terms, one gets :

$$\left\langle f_s^{-1}\right\rangle = \left[\sum_i\frac{\left\langle\left(c_i^{(eq)}\right)^2\right\rangle}{R_i^2} - \frac{\left\langle c_x^{(eq)}c_y^{(eq)}\right\rangle}{R_x R_y}\right]^{1/2} \tag{33}$$

$$\left\langle f_d^{-1}\right\rangle = \left[\sum_i\frac{\left\langle\left(a_i^{(eq)}\right)^2\right\rangle}{F_i^2} - \frac{\left\langle a_x^{(eq)}a_y^{(eq)}\cos\left(\varphi_y-\varphi_x\right)\right\rangle}{F_x F_y}\right]^{1/2} \tag{34}$$

Hence the fatigue "safe-life" design criterion for anisotropic metal elements under multiaxial static and dynamic random loads becomes as follows :

$$\begin{aligned}&\left(\sum_i\frac{\left\langle c_i^2\right\rangle}{R_i^2} - \frac{\left\langle c_x c_y\right\rangle}{R_x R_y}\right)^{1/2} +\\&+\left\{2\int_{-\infty}^{\infty}\left[\sum_i\frac{S_i(\omega)}{F_i^2} - \frac{S_{x,y}(\omega)}{F_x F_y}\right]d\omega\right\}^{1/2} < 1\end{aligned} \tag{35}$$

CONCLUSIONS

- The fatigue "safe-life" design criterion which covers the conditions of both static strength and fatigue safety of metal elements under multiaxial static and dynamic random loads, was formulated.
- The presented criterion includes material constants which :
  - have simple physical interpretation
  - can be determined by uniaxial tests
  - are directly related to the applied load
  - and can reflect material anisotropy.

NOMENCLATURE

- a_i - amplitude of i-th component of the in-phase stress (i = x, y, z, xy, yz, zx)
- $a_i^{(eq)}$ - random amplitude of i-th equivalent stress component
- a_{i1}, a_{i2} - quantities defined by (5)
- c_i - mean value of i-th component of the in-phase stress, random mean value of i-th component of the actual stress
- $c_i^{(eq)}$ - random mean value of i-th equivalent stress component
- E - Young modulus
- F_i - fatigue limit under fully reversed load relevant to the stress amplitude a_i
- f_d, f_s - partial safety factors acc. to [1]
- j - imaginary unit
- K_i - autocorrelation function of the process $s_i(t)$
- $K_{x,y}$ - cross-correlation function of the processes $s_x(t)$ and $s_y(t)$
- R_i - yield strength relevant to the mean stress value c_i
- s_i - time-variable part of i-th stress component
- $s_i^{(eq)}$ - time-variable part of i-th equivalent stress component
- S_i - power spectral density of the process $s_i(t)$
- $S_{x,y}$ - cross power spectral density of the processes $s_x(t)$ and $s_y(t)$
- t - time

T	-	averaging time
δ	-	Dirac's delta function
ν	-	Poisson's ratio
σ_i	-	i-th stress component
$\sigma_i^{(eq)}$	-	i-th equivalent stress component
τ	-	time interval
φ_i	-	random phase angle of i-th equivalent stress component
ϕ, ϕ_{eq}	-	strain energy of distortion per unit volume in the actual and equivalent stress states
ω	-	circular frequency
ω_{eq}	-	equivalent circular frequency
$\langle \bullet \rangle$	-	expected value
$(\bullet)^*$	-	complex conjugate

BIBLIOGRAPHY

1. Kolenda J.: *Fatigue „safe-life" criterion for metal elements under multiaxial constant and periodic loads.* Polish Maritime Research, Vol. 11, No 2, June 2004
2. Łagoda T., Macha E.: *Estimated and experimental fatigue lives of 30CrNiMo8 steel under in- and out-of-phase combined bending and torsion with variable amplitudes.* Fatigue Fract. Engng Mater. Struct., No 11, 1994
3. Macha E., Sonsino C. M.: *Energy criteria of multiaxial fatigue failure.* Fatigue Fract. Engng Mater. Struct., Vol. 22, 1999
4. Sonsino C. M.: *Multiaxial fatigue of welded joints under in-phase and out-of-phase local strains and stresses.* Int. J. Fatigue, No 1, 1995
5. Troost A., El-Magd E.: *Schwingfestigkeit bei mehrachsiger Beanspruchung ohne und mit Phasenverschiebung.* Konstruktion, No 8, 1981
6. Palin-Luc T., Lasserre S.: *High cycle multiaxial fatigue energetic criterion taking into account the volumic distribution of stresses.* Proc. of the 5th Int. Conf. on Biaxial /Multiaxial Fatigue and Fracture, Opole. Vol. 1, 1997
7. Blake A. (Ed.): *Handbook of mechanics, materials and structures.* J. Wiley & Sons. New York, 1985
8. Życzkowski M. (Ed.): *Technical mechanics.* Vol. 9, Strength of structural elements (in Polish). PWN. Warszawa, 1988
9. Pitoiset X, Preumont A.: *Spectral methods for multiaxial random fatigue analysis of metallic structures.* Int. J. Fatigue, Vol. 22, 2000
10. Preumont A.: *Vibrations aléatoires et analyse spectrale.* Presses Polytechniques et Universitaires Romandes, CH-1015. Lausanne, 1990

CONTACT WITH THE AUTHOR

Janusz Kolenda, Prof, D.Sc.
Faculty of Ocean Engineering
and Ship Technology,
Gdańsk University of Technology
Narutowicza 11/12
80-952 Gdańsk, POLAND
e-mail : sek7oce@pg.gda.pl

Miscellanea

TOP KORAB activity in 2003

In 2003 The Polish Society of Naval Architects and Marine Engineers, TOP KORAB, – in accordance with its custom of arranging topical meetings – carried out, in Gdańsk, 9 meetings devoted to the following topics :

- Achievements of POL-LEVANT shipping company in the light of situation of Polish shipping
- Development stategy of Pomeranian region
- Certification of management systems
- Dimension allowance system applied in shipbuilding industry
- Current problems of Central Maritime Museum in Gdańsk - presentation of permanent exhibitions
- Achievements and development prospects of Gdynia Naval Shipyard
- Prospects and future of Polish Shibuilding Industry Forum
- A proposal of topics of club meetings to be organized in 2004, and of forms of future activities of TOP KORAB
- Presentation of the current state of TOP KORAB chronicle.

A full-day coach excursion to Malbork to visit the historical Teutonian Knights' castle, was also carried out.

Whereas in Szczecin, apart from 4 organizational meetings, 2 topical meetings were held :

- Current problems of NOWA Szczecin Shipyard Co
- Problems associated with implementing a new IMO code : ISPS Code 2003.

Ministerial award

Some years ago a team of the Faculty of Ocean Engineering and Ship Technology, Gdańsk University of Technology, has commenced working on design and construction of a prototype fishing cutter for Baltic Sea, which has been hoped to be useful in renewal of the obsolete fishing vessels flying Polish flag. The undertaken task was sucessfully completed and the built cutter has already operated for a year, gathering flattering opinions from the side of its users.

Polish state authorities received with recognition that important achievement seeing in it a nucleus of a new generation of ecolgical family-operated fishing vessels. Giving voice to it, the authorities rewarded the team consisted of the following persons :

- K. Rosochowicz, Prof.D.Sc., Eng.
- A. Wołoszyn, M.Sc., Eng.
- J. Krępa, D.Sc., Eng
- Cz. Dymarski, D.Sc., Eng.
- T. Blekiewicz, Eng.
- E. Brzoska, D.Sc., Eng.
- G. Wendt, M.Sc., Eng.
- M. Stachowiak, Eng.

with the Ministerial prize which was solemnly handed over on 13 May 2004.

Physical aspects of application and usefulness of semi-Markovian processes for modelling the processes occurring in operational phase of technical objects

Jerzy Girtler
Gdańsk University of Technology

ABSTRACT

In the paper usefulness of semi-Markovian processes for modelling real processes which occur in the operational phase of various technical objects, is considered. The usefulness of this theory was proved by presenting specific features of the semi-Markovian processes and physical aspects of their use as models of the processes occurring in the operational phase of the objects in question. The specific features were described with taking into account the process of changing the technical states of such objects. The physical aspects of models in the form of semi-Markovian processes were justified by postulating a hypothesis by which wear process of tribological units of crucial sub-assemblies of machines, can be explained. Also, the consequences logically resulting from the hypothesis and necessary to its verification, are discussed. A method for its verification is also attached. It was proved possible to model the real processes in question by means of the continuous semi-Markovian processes of finite set of states.

Key words : technical object, semi-Markovian process, tribological unit, wear

INTRODUCTION

The theory of semi-Markovian processes gains more and more widespread applications in engineering sciences and in operation of technical objects (e.g. diesel engines, gas turbines, screw propellers, pumps, compressors, coolers, filters, ship propulsion systems). It makes it possible to elaborate models of various real processes, including processes of changes of technical and operational states as well as operational processes of any technical objects.

The models are elaborated in the form of special stochastic processes, i.e. semi - Markovian ones, and recently also decision-related (controlled) semi-Markovian processes whose realizations depend on decisions made in the instants of changing their states [4, 6, 8, 11]. However it is not easy to apply the processes because of their specific features. Not taking them into account may result in building such models of real processes in the form of semi-Markovian ones whose investigations cannot provide any new information about a modelled process, regarding e.g. durability and reliability of technical objects, their load spectra etc. Therefore it is worthwhile to indicate the physical aspects of application of semi-Markovian processes as models of real processes related to technical objects, at least with taking as an example the process of changing technical states during operation of such objects.

SPECIFIC FEATURES OF SEMI-MARKOVIAN PROCESSES

The semi-Markovian processes are stochastical ones of peculiar features. In the literature there are different definitions of semi-Markovian process, which have different ranges of generality and exactness. For purposes of the modelling of operation of technical objects the semi-Markovian process (family of random variables)

$$\{Y(t) : t \in T\} \text{ at } T = [0 , + \infty]$$

can be defined by means of the so - called uniform Markovian renewal process. Such definition proposed by F. Grabski [9] is close to those given by other authors [13, 19, 20, 1].

From the definition it results that it is stochastic one of a discrete set of states and its realizations are right-hand continuous functions constant within intervals (of uniform values within operational time intervals which are random variables). Such process is defined only when its initial distribution $P_i = P\{Y(0) = s_i\}$ as well as the functional matrix $\mathbf{Q}(t) = [Q_{ij}]$ is known; the matrix elements are the probabilities of transition from the state s_i to the state s_j, within the time not greater than t ($i \neq j$; i, j = 1, 2,..., k), being the non-decreasing functions $Q_{ij}(t)$ of variable t, namely :

$$Q_{ij}(t) = P\{Y(\tau_{n+1}) = s_j \ , \ \tau_{n+1} - \tau_n < t \big| Y(\tau_n) = s_i\} \quad (1)$$

A semi-Markovian model of an arbitrary real process can be formed only when states of the process can be defined in such a way that duration time of a state appearing in the instant τ_n as well as a state possible to be reached in the instant τ_{n+1} do not depend stochastically on the preceding states and their duration time intervals.

For elaborating the semi-Markovian model $\{W(t) : t \in T\}$ of a given real process it is necessary to apply the theory of semi-Markovian processes. It makes it possible to determine probabilistic characteristics of an arbitrary random process (if only such model is formed), which may be of a practical importance. Such process may be a real process of changing technical states of any technical object. An example model of such process can be presented in the form of a graph of changing the states of the object (Fig.1). The model is one of the crucial components of the operational model of every technical object. One of the possible realizations of the process of changing the technical states, $\{W(t) : t \in T\}$, is presented in Fig. 2.

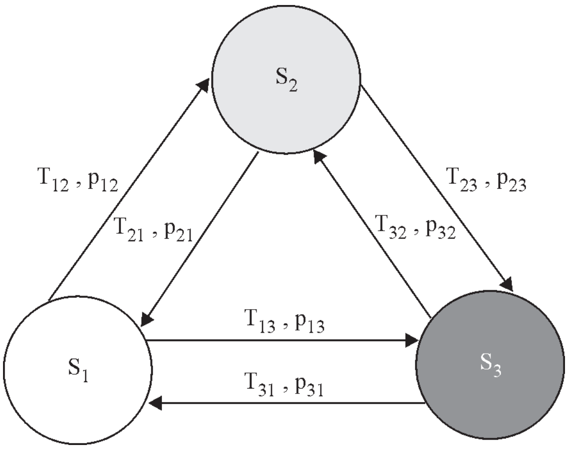


Fig. 1. *A graph of changing the states of an arbitrary technical object : s_1 - full serviceability state, s_2 - partial serviceability state, s_3 - unserviceability state, p_{ij} - probability of passing the process from the state s_i to the state s_j , T_{ij} - duration time of the state s_i provided the process passed to the state s_j ; i, j = 1, 2, 3*

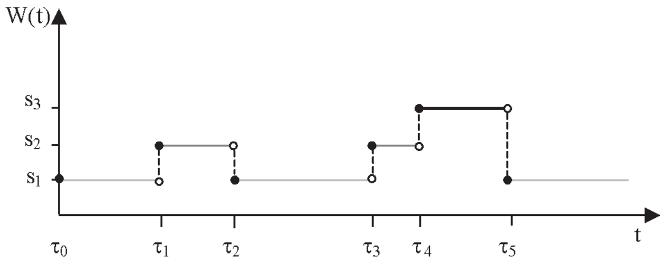


Fig. 2. *An example realization of the process $\{W(t) : t \in T\}$ of an arbitrary technical object : $\{W(t): t \in T\}$ - process of changing the technical states, t - time of operation; s_1 - state of full serviceability, s_2 - state of partial serviceability, s_3 - state of unserviceability*

In reality the process of changing the technical states of any technical object is stochastic, continuous in time and throughout the states. It means that the realizable kinds of technical state form a set of technical states of a given object, which is infinite one.

Identification of all technical states of many technical objects (e.g. combustion engines, boilers, pumps, compressors, ship prpulsion systems) is neither possible nor purposeful, both due to technical and economical reasons. Hence it is necessary to split the set of states of technical objects into a few number of classes (sub-sets) of technical states.

Assuming, as a splitting criterion, usefulness of a given technical object for realization of tasks (serviceability) one can distinguish the following classes (sub-sets) of technical states, shortly called "states" [4, 6, 8] :

- ★ the state of full serviceability, s_1 , which makes it possible to use the object in any conditions and in any range of loads, to which it was adjusted in the phases of its designing and manufacturing
- ★ the state of partial serviceability, s_2 , which makes it possible to use the object in limited conditions and in a range of loads lower than those to which it was adjusted in the phases of its designing and manufacturing
- ★ the state of unserviceability, s_3 , which does not make it possible to use the object in accordance with the purpose for which it was intended (e.g due to its failure, carrying out maintenance operations on its subassemblies etc.).

Therefore it is a three-state process
of continuous realizations (continuous with time).

The technical objects which are in the full serviceability state (s_1) can be used in any instant and conditions to which it was adjusted in the designing and manufacturing phases, and under various loads. And, the technical objects which are in the partial serviceability state (s_2) can be used or maintained depending on a decision-making situation (control strategy), whereas the unserviceable technical objects (in the state s_3) due to their failure are always maintained provided it would be cost-effective. However the technical objects unserviceable because of realization of preventive maintenance, are unserviceable only during the time of realization of these operations which require the object's structure to be trespassed by disassembling some of their devices.

The particular states $s_i \in S$ (i = 1, 2, 3) can be identified by means of an appropriate diagnostic system (SD) whose usefulness depends on quality of an applied diagnosing system (SDG) as well as on its capability of identifying the states of the technical object considered as a diagnosed system (SDN).

It can be assumed that if whichever of the states s_2 or s_3 does not occur then the object in question is in the state s_1.

The considered process of changing the technical states of technical object is, in mathematical terms, a function which maps the set of the instants T into the set of technical states, S. Such process can be modelled by means of the stochastic processes of discrete set of states and continuous duration time of distinguished technical states of the object.

Therefore the set of technical states :

$$S = \{s_1, s_2, s_3\}$$

can be considered as the set of values
of the stochastic process

$$\{W(t) : t \in T\}$$

whose realizations are constant within time intervals
and right-hand-side continuous, Fig.2.

In the case of the process of operation of technical objects the following characteristics may be of a practical importance :

- ➢ the one-dimensional distribution of the process (instantaneous distribution), whose elements are the functions $P_k(t)$ representing the probability of the event that in the instant t the process will enter the state s_k
- ➢ the limiting distribution of the process $P_j = \lim_{t \to \infty} P\{Y(t) = s_j\}$
- ➢ the conditional probabilities, i.e. those of transition of the process from the state s_i to the state s_j ,
  $P_{ij}(t) = P\{Y(t) = s_j / Y(0) = s_i\}$
  (transition probabilities)

- the distribution of the time of the first transition of the process from the state s_i to the sub-set of the states A ($\Phi_{iA}(t)$), and if this sub-set contains only one element – to the state s_j, i.e. the distribution $\Phi_{ij}(t)$
- the distribution of return time of the process to the state s_j, i.e. the distribution $\Phi_{jj}(t)$
- the asymptotic distribution of the renewal process $\{V_{ij}(t) : t \geq 0\}$, generated by return time intervals of semi-Markovian process (to the state s_j available from the state s_i), which at the instant t, obtains a value equal to number of "coming-in" events of that process to the state s_j
- an approximate distribution of the total time of maintaining the process Y(t) in the state s_j provided the state s_i is that initial one
- the expected value $E(T_i)$ of the duration time T_i of the state s_i of the process irrespective of the state to which transition occurs at the instant τ_{n+1}
- the variance $D^2(T_i)$ of the duration time T_i of the state s_i
- the expected value $E(T_{ij})$ of the duration time T_{ij} of the state s_i of the process provided the state s_j is the next one
- the expected value $E(\Theta_{jj})$ of the random variable Θ_{jj} which represents the return time of the process to the state s_j
- the expected value $E\{V_{ij}(t)\}$ of the random variable $V_{ij}(t)$ which represents the number of "coming-in" events of that process to the state s_j within the time interval [0 , t]
- the variance $D^2\{V_{ij}(t)\}$ of the random variable $V_{ij}(t)$
- the average number of "coming-in" events, $\lambda_{ij}(t)$, of the process to the state s_j, related to a unit of time, provided the state s_i of the process is that initial one (i.e. the intensity of the " coming-in" events of the process to the state s_j provided $Y(0) = s_i$)
- the limiting intensity of the "coming-in" events of the process to the state s_j, i.e. the intensity $\lambda_{ij} = \lim_{t\to\infty}\lambda_{ij}(t)$.

To obtain numerical values of the above mentioned characteristics is possible if two following conditions will be satisfied :

- if appropriate statistical data whose values would represent estimation of the transition probability p_{ij}, of the expected value $E(T_i)$, etc, will be collected
- if a semi-Markovian model of operation process of technical objects having a small number of its states and mathematically simple functional matrix **Q**(t) will be elaborated.

The second condition is important in the case of calculating the instantaneous distribution of states of the process $P_k(t)$. The distribution can be calculated if the initial distribution of the process and its functions $P_{ij}(t)$ are known. The calculating of the probabilities $P_{ij}(t)$ consists in solving the set of Volterra second-kind equations in which the functions $Q_{ij}(t)$ being elements of the process functional matrix **Q**(t), are known quantities [9].

In the case when number of process states is small and the functional matrix of the process – rather simple, that set can be solved by using the Laplace transform method [9, 10, 20]. However when number of process states is large or when its functional matrix (core of the process) is very complex, only an approximate solution of the set of the equations is available. Such (numerical) solution does not make it possible to determine values of probabilities of occurrence of process particular states if t is of a large value (theoretically if $t \to \infty$). The numerical solution does not provide any answer to the question very important for operational practice, namely : in which way do the probabilities of semi-Markovian process states change if t is large?

From the semi-Markovian process theory it results that the probabilities, in the case of the ergodic semi-Markovian processes, tend along with time to strictly determined constant numbers. They are called the limiting probabilities of states and their sequence forms the limiting distribution of the process. The distribution makes it possible to define the availability factor of technical object as well as the income or cost per unit time of its operation. The quantities serve as criterion functions in solving problems of operation process optimization of technical objects. Such distribution can be calculated much easier than the instantaneous one.

Similar difficulties are associated with solving the set of integral equations which make it possible to calculate the distribution of the random variable Θ_{iA} which determines the time of the first transition of the process from the state s_i to the sub-set of states, A, or (if the state A contains only one element) – to the random variable Θ_{ij} which determines the time of the first transition of the process from the state s_i to the state s_j.

PHYSICAL ASPECTS OF APPLICATION OF SEMI-MARKOVIAN PROCESSES FOR MODELING THE PROCESSES OCCURRING IN THE PHASE OF OPERATION

From the presented considerations it results that the semi-Markovian models are characteristic of the following features [7, 9, 10, 19, 20] :

- Firstly, is satisfied the Markov condition that future evolution of the investigated object (e.g. the process of changing the technical states in the object's operational phase), for which the semi-Markovian model has been elaborated, should depend only on its state at a given instant but not on the functioning of the object in the future, i.e. that the *future* of the object would not depend on its *history* but only on its *present*.
- Secondly, the random variables T_i (determining duration time of the state s_i regardless which state will occur after it) as well as T_{ij} (determining duration time of the state s_i provided the next state of the process will be the state s_j) have their distributions different from exponential ones.

Therefore in the modeling aimed at elaboration of a semi-Markovian model of the process of changing the object's technical states, an analysis of changing the states of real process, i.e. those occurring in the operational phase of the object in question, should be taken into account.

In the case of every technical object the process of changing its technical states is that where the duration time intervals of each its state are random variables. Particular realizations of the random variables depend on many factors, a.o. on the technical object's wear.

In the case of such technical objects as e.g. diesel engines, compressors or pumps, it was observed that wear of their sliding tribological units is weakly correlated with time [3, 8, 16, 17, 23, 24]. The observation is important because serviceability of such machines depends mainly on the technical state (i.e. on wear) of their tribological units. This made it possible to predict technical states of such machines with taking into account solely their present state and neglecting those earlier occurred. An explanation of the fact would make it possible to elaborate (by applying the theory of semi-Markovian processes) more adequate mathematical probabilistic models for predicting the technical states of particular machines. To this end the following hypothesis (H) can be offerred :

A state of an arbitrary sliding tribological unit as well as its duration time essentially depend on the state preceding it and neither on the earlier occurred ones and nor their duration time intervals because its load and both rate and increments of wear, implicated by it, are the processes of asymptotically independent values.

The last statement of the hypothesis (because its load...) results from two obvious facts :

- there is a strict relationship between loading on sliding tribological units and their wear [15, 17, 23]
- there is a lack of monotonic changes of loading on tribological units of machines within longer time of their operation, hence their loading can be assumed stationary [3, 17, 21, 22, 23].

The load stationarity (in a broader sense) means in every case that all multi-dimensional probability density functions depend only on mutual distances of the instants $\tau_1, \tau_2, ..., \tau_n$, and not on their values [5]. Therefore the one-dimensional probability density function of load values does not depend on the instant related to a given value, and the two-dimensional probability density function depends only on difference of the instants in which observed loading values occurr.

And, in a narrower sense, the fully stationary loading is understood as that whose all possible statistical moments of higher orders as well as the total moments of loading considered as a process, are not time - dependent. In the case of the fully stationary process (*in a narrower sense*) its characteristic quantities are as follows :

the expected value $m(t) = m = const$
variance $D^2(t) = \sigma^2 = const$
autocorrelation $A(\tau_1, \tau_2) = A^*(\tau_2 - \tau_1) = A^*(r)$
and autovariance $K(\tau_1, \tau_2) = K^*(\tau_2 - \tau_1) = K^*(r)$.

However the stationary process *in a broader sense* is characteristic of :

$m(t) = m = const$ as well as $A(\tau_1, \tau_2) = A^*(\tau_2 - \tau_1) = A^*(r)$.

In practice the loading stationarity in a broader sense is important. And, in this case to investigate the loading on tribological units in order to reveal the enumerated properties, is not necessary, as it is known from investigations of different machines, which have been performed so far, that the loading on their tribological units continously changes in such a way that its particular values measured after very short time intervals are strongly correlated to each other. However when the time interval between measurements of loads increases, the correlation between the loads decreases. Therefore the loading values measured in the time intervals (or instants) very distant apart can be considered as independent ones. This feature is called the asymptotic independence of a loading value measured in the instant, e.g. τ_{i+1}, from that measured in the instant τ_i, when the range $\Delta\tau = \tau_{i+1} - \tau_i$ is large enough.

The so understood asymptotic independence between loading values either measured or calculated in the instants τ_i and τ_{i+1}, is manifested by that their mutual dependence decreases along with increasing the range $\Delta\tau$. Moreover, from work principles of particular machines it is also known that their loading considered within a longer time of their correct operation, does not (and cannot) show any monotonically increasing or decreasing changes. Therefore one can assume that the maximum loading values appear in the specified instants accidentally, always with some probability only. This lack of monotonicity of the loading is called its stationarity.

In order to verify the presented hypothesis (H) it is necessary to predict the consequences whose occurrence can be confirmed empirically if the hypothesis is true. The consequences (K) which can be derived from the hypothesis (with taking into account the mentioned features of loading on ship power plant machines and their sliding tribological units) are the following [3] :

- K_1 - irregular course of realization of wear process of particular sliding tribological units
- K_2 - interweaving realzations of wear processes of sliding tribological units
- K_3 - such course of autocorrelation function for a given sliding tribological unit that at first the function fast decreases along with the range $\theta = h\Delta\tau$ (h = 1, 2,..., n) increasing, and next it oscillates about zero, at relatively small amplitude smaller and smaller along with $\Delta\tau$ increasing
- K_4 - almost normal distribution of wear increments of sliding tribological units for a sufficiently long time interval (Δt) of their correct operation
- K_5 - linear relationship of variance of wear process of sliding tribological units and their operation time values.

The above described consequences are graphically illustrated in Fig. 3 ÷ 6.

The presented consequences can be justified as follows :

If the features of the loading on machines and thus their tribological units are such as those above described, then the course of realization of wear of the units will have to be irregular. This forms the basis to assume that the wear increments recorded in the time intervals much distant from each other,

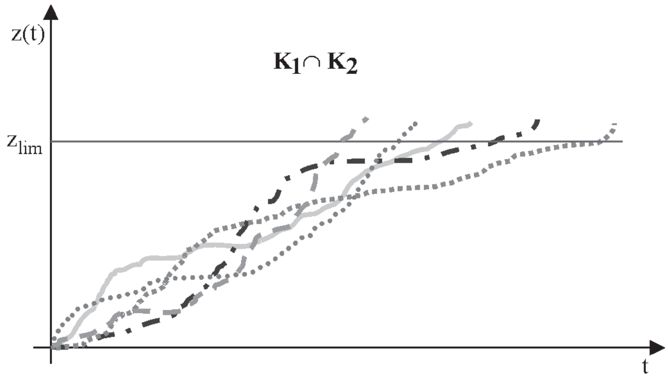


Fig. 3. *Example realizations of wear processes of sliding tribological units : z - wear, z_{lim} - limiting wear value, t - object's operation time*

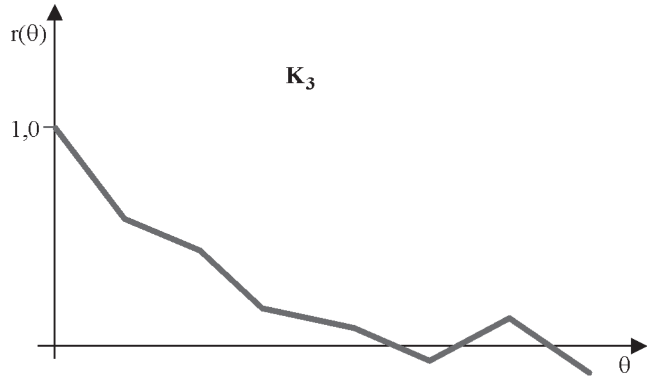


Fig. 4. *Example course of the autocorrelation function r(θ) where : θ - the range between time intervals (within which wear was investigated)*

are asymptotically independent, and along with increasing the time (time range θ, where : $\theta = h\Delta\tau$, h = 1, 2,..., n) between the intervals the relationship between the increments in question will be weakening. Hence the wear processes of such units can be considered as those of asymptotically independent increments [3].

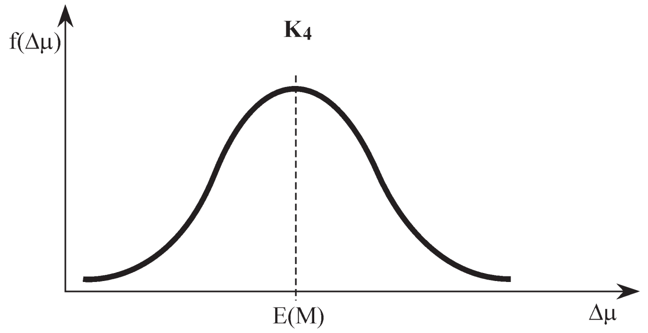


Fig. 5. *Example form of the density function f (Δμ) of the asymptotically independent wear increments Δμ of sliding tribological units, for sufficiently long time interval (Δt) of their correct operation*

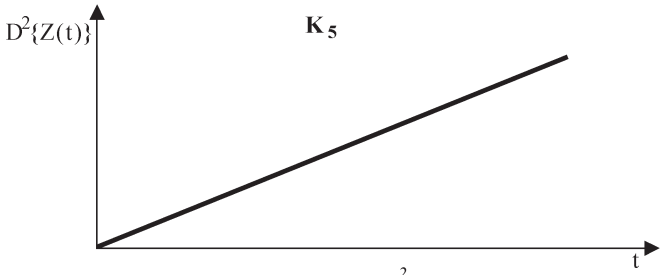


Fig. 6. *Example relationship of the variance $D^2\{Z(t)\}$ of the wear process Z(t) of sliding tribological units of asymptotically independent increments, and their operation time t*

One of the most important characteristics of every stochastic process (including wear processes) is the autocorrelation function $r(\theta) = f(\theta)$, where : r - autocorrelation coefficient, θ - time range. If $r(\theta)$ decreases then wear process of a given tribological unit can be considered as that of asymptotically independent increments. Hence for tribological units in question an almost normal distribution of wear increments should be expected in a sufficiently long time interval. Moreover, if wear process is of asymptotically independent increments its variance $D^2\{Z(t)\}$ increases linearly along with correct operation time, in accordance with the relationship (2) :

$$D^2\{Z(t)\} = At + B \tag{2}$$

where :
A and B - process constants

The above mentioned consequences $K_i (i = 1, 2,..., 5)$ reveal the probabilistic principle of wear of sliding tribological units. They are not contradictory to each other, and their logical veracity is doubtless. Hence it is possible to consider all of them as one consequence K and to use it for empirical proving if the presented hypothesis (H) is true or false. Such verification consists in experimental testing of wear of sliding tribological units and checking if the consequence K is true, which is equivalent to checking if the consequences K_i (the facts) appear or do not appear. Such verification of the hypothesis H makes it necessary to accept the veracity of the following syntactic implication [8, 12, 18] :

$$H \Rightarrow K \tag{3}$$

Then the non-deductive (inductive) reasoning may be used in accordance with the following scheme [8 , 18] :

$$(K , H \Rightarrow K) \vdash H \tag{4}$$

where :
$K = \{K_i , i = 1, 2,..., 5\}$

Its logical interpretation is as follows : if the empirical testing of the consequence K confirmed its veracity, then if the implication (3)is true, the hypothesis H is also true and acceptable. The reasoning in accordance with (4), called reductive one, does not lead to any firm conclusions but only to probable ones [12 , 18].

Semi-Markovian model of an arbitrary operational process can be applied in the case of machines even by using the diagnostic systems (SDG) where the reductive reasoning can be applied. Hence when reasoning on a diagnosis (conclusion) concerning the state of a given technical object being a diagnosed system (SDN) the statement K (stating that this – and not another – vector of values of diagnostic parameters is observed) is deemed to be a fully firm premise.

However the statement S (stating that there is this – and not another – state of SDN) is a conclusion formulated on the basis of the statement K during the non-deductive reasoning process which proceeds in compliance with the following scheme :

$$(K , S \Rightarrow K) \vdash S \tag{5}$$

where :
K - fully firm premise
S - conclusion formulated on the basis of the statement K.

From such reasoning the following hypothesis results :
The considered SDN is in the state S because the vector of values of diagnostic parameters, K, is observed.

It can be also formulated in another, equivalent way :
The vector of values of diagnostic parameters, K, is observed because the SDN in question is in the state S.

Such reasoning does not make it possible to formulate firm conclusions but only probable ones. Therefore it is not possible to exactly determine a technical state of a diagnosed system (SDN) and thus to control its operational process in such a way that a future state would depend on many previous states.

FINAL REMARKS AND CONCLUSIONS

- The semi-Markovian processes are useful models for investigating the real processes which occur in the operational phase of technical objects. Hence elaboration of a semi--Markovian model of a given process occurring in operational phase of an arbitrary technical object, makes it possible to easily determine probabilistic characteristics of the process in question.
- In practice the semi-Markovian processes are more useful than Markovian ones. It means that the semi-Markovian processes of continuous time parameter and finite set of states are characteristic of that the time intervals of staying the processes in particular states are random variables of arbitrary distributions concentrated in the set $R_+ = [0 , \infty]$. This differs them from the Markovian processes whose intervals are random variables of exponential distributions.
- A semi-Markovian model of an arbitrary process occurring in the phase of operation of a technical object is that of a finite set of states and continuous time.
- An additional benefit from application of semi-Markovian processes (like also of Markovian ones) is the possibility of using professional computer tools for solving different sets of state equations for such models of real processes.

- ❖ Semi-Markovian model of an arbitrary operational process can be applied in the case of machines even by using the diagnostic systems (SDG) where the reductive reasoning can be applied. However such reasoning does not make it possible to formulate firm conclusions but only probable ones.

BIBLIOGRAPHY

1. Cinlar E.: *Markov renewal theory: a survey*. Mag. Sci. No. 7, 1975
2. Girtler J.: *A model of ship diesel engine operation* (in Polish). Zagadnienia Eksploatacji Maszyn, iss. 4/1988
3. Gercbach I. B., Kordoński CH.B.: *Reliability models of technical objects* (in Polish). WNT. Warszawa, 1968
4. Girtler J.: *A stochastic model of ship diesel engine operational process* (in Polish). Zagadnienia Eksploatacji Maszyn, iss. 2/1989
5. Girtler J.: *Semi-Markov model of changes of sea-ships' and aircrafts' moving*. Archives of Transport, Vol. 11, iss.1-2/99
6. Girtler J., Kuszmider S., Plewiński L.: *Selected problems of operation of sea-going ships in the aspect of safety at sea* (in Polish). Monography. Maritime University, Szczecin, 2003
7. Girtler J.: *Possible application and usefulness of semi--Markovian models of machine operation processes* (in Polish). Zagadnienia Eksploatacji Maszyn, iss. 3/1996
8. Girtler J.: *Diagnostics as a condition for controlling of operation of ship diesel engines* (in Polish). Maritime University. Studies No. 28. Szczecin, 1997
9. Grabski F.: *Theory of semi-Markovian processes of operation of technical objects* (in Polish). Polish Naval University. Scientific bulletin, No. 75 A. Gdynia, 1982
10. Grabski F.: *Semi-Markovian models of reliability and operation* (in Polish). IBS PAN. Monography. Series: *System research*, Vol. 30. Warszawa, 2002
11. Howard R.A.: *Research in semi-Markovian decision structures*. J. Oper. Res. Soc. Japan, 1964, Vol. 6, No. 4
12. Leszek W.: *Empirical investigations* (in Polish). Institute of Operation Technology. Radom, 1977
13. Koroluk V.S., Turbin A.F.: *Fazovoje ukrupnenia sloznych sistem*. Wyssaja Skola. Kijev, 1978
14. Mine H., Osaki S.: *Markovian decision processes*. AEPCI. New York, 1970
15. Niewczas A.: *Background for stochastic model of frictional wear in durability problems of machine elements* (in Polish). Radom University of Technology. Scientific bulletin, No. 19. Radom, 1989
16. Niziński S.: *Operation of technical objects* (in Polish). Institute of Operation Technology. Radom, 2002
17. Piotrowski I.: *Ship combustion engines* (in Polish). Wydawnictwo Morskie. Gdańsk, 1983
18. Pabis S.: *Metodology and methods of empirical sciences* (in Polish). PWN. Warszawa, 1985
19. Pyke R.: *Markov renewal processes with finitely many states*. Ann. Math. Statist. 1961, Vol. 32, No. 4
20. Silvestrov D.S.: *Polumarkovskije processy s diskretnym mnozestwom sostojanij*. Sovetskoe Radio. Moskva, 1980
21. Sołdek J.: *Automation of ships* (in Polish). Wydawnictwo Morskie. Gdańsk, 1985
22. Wajand J.A.: *Self-ignition engines* (in Polish). WNT. Warszawa, 1988
23. Zwierzycki W. (Editor) : *Selected problems of material wearing in sliding units of machines* (in Polish). PWN. Warszawa--Poznań, 1990
24. Zwierzycki W.: *Prediction of reliability of wearing elements of machines* (in Polish). Institute of Operation Technology. Radom, 2000

CONTACT WITH THE AUTHOR

Jerzy Girtler, Prof., D.Sc.
Faculty of Ocean Engineering
and Ship Technology,
Gdańsk University of Technology
Narutowicza 11/12
80-952 Gdańsk, POLAND
e-mail : jgirtl@pg.gda.pl

Conference

Plenary session of Machine Building Committee held in coast region

On 31 May and 1 June 2004 members of the Machine Building Committee, Polish Academy of Sciences held, this turn in Gdańsk, their plenary session commencing the next tenure of activity of the Committee. It was devoted, apart from organizational matters, to new challenges arising from the entrance of Poland into European Union.

On this occasion took place an open scientific meeting on :

Development prospects of machine building and operation after entrance of Poland into European Union

8 papers were prepared to be presented during the meeting :

- *Poland in the European Space of Science and Education* by Prof. W. Sadowski (Gdańsk University of Technology)
- *Polish Space of Science − prospects of development* by Prof. J. Kiciński (Insitute of Fluid Flow Machinery, Polish Academy of Sciences, Gdańsk)
- *Application trends in EU programs in the area of machine building and operation* − by Prof. A. Mazurkiewicz (Institute of Technical Operation Processes)
- *Contemporary research and education problems of ocean engineering* − towards European Space of Scientific Research − by Prof. J. Szantyr (Faculty of Ocean Engineering and Ship Technology, Gdańsk University of Technology)
- *Proecological activity of Mechanical Faculty concerning machine building and operation, in the frame of EU* by Prof. W. Przybylski (Mechanical Faculty, Gdańsk University of Technology)
- *Attempts to problems of operation of ship engines in the aspect of cooperation in the frames of EU and NATO* by Prof. R. Cwilewicz (Gdynia Maritime University), Prof. L. Piaseczny (Polish Naval University, Gdynia)
- *Integration of doctorate studies* − by Prof. B. Żółtowski (Technical Agricultural Academy, Bydgoszcz)
- *MARIE CURIE individual grants* − basic information by Prof. W. Zwierzycki (Poznań University of Technology)

The two-day meeting was co-organized by the Institute of Fluid Flow Machinery, Polish Academy of Sciences, Mechanical Faculty and Faculty of Ocean Engineering and Ship Technology, Gdańsk University of Technology.

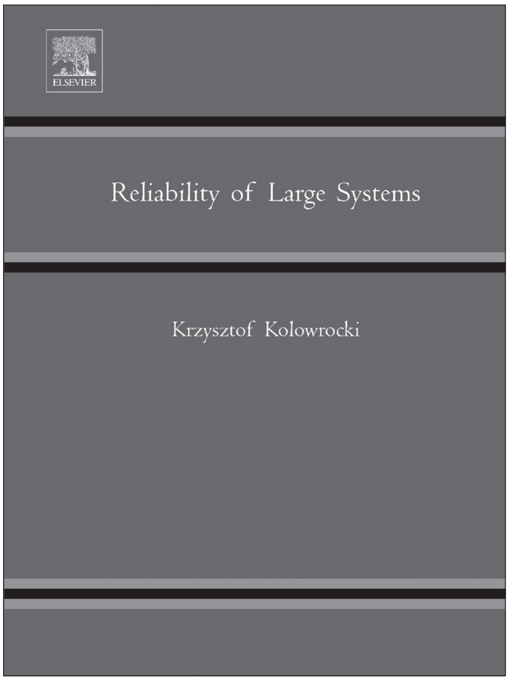
Reliability of Large Systems

An interesting book written by :

Krzysztof Kołowrocki
Gdynia Maritime University,

has been recently published by Elsevier (April 2004).

The Book Website is : http://www.elsevier.com/wps/find/bookdescription.cws_home/702918/description



The book consists of eight chapters whose content is shortly characterized below.

1. Basic notions

Basic notions and agreements which are necessary to further considerations, are introduced. The asymptotic approach to the system reliability investigation and the system limit reliability function is defined.

2. Two-state systems

Two-state homogeneous and non-homogeneous series, parallel, "m out of n", series-parallel and parallel-series systems are defined. Their exact reliability functions are determined.

3. Multi-state systems

Basic notions of the system multi-state reliability analysis are introduced. The multi-state homogeneous and non-homogeneous series, parallel, "m out of n", series-parallel and parallel-series systems with degrading components are defined and their exact reliability functions are determined. The multi-state limit reliability function of the system, its risk function and other multi-state system reliability characteristics are introduced and determined.

4. Reliability of large two-state systems

Auxiliary theorems on limit reliability functions of large two-state systems, which are necessary for their approximate reliability evaluation, are formulated. The classes of limit reliability functions for homogeneous and non-homogeneous series, parallel, series-parallel and parallel-series systems and for a homogeneous "m out of n" system are established.

Applications of the asymptotic approach to reliability evaluations of model systems are presented. Six corollaries are formulated and proved on the basis of the auxiliary theorems and applied to finding limit reliability functions of the considered systems and approximate evaluations of their reliability functions, lifetime mean values and lifetime standard deviations.

The reliability evaluation is done for the following systems: the model non-homogeneous series system, the homogeneous parallel system of an energy cable, the "16 out of 35" lighting system, the homogeneous regular series-parallel gas distribution system, the non-homogeneous regular series-parallel water supply system and the model homogeneous regular parallel-series system.

The accuracy of the performed evaluations is illustrated in tables and figures. The reliability data of components are assumed either to be arbitrary or to come from experts. These reliability evaluations of the considered systems' characteristics are an illustration of the possibility of applying the asymptotic approach to system reliability analysis of large real technical systems.

5. Reliability of large multi-state systems

Auxiliary theorems on limit reliability functions of multi-state systems, which are necessary for their approximate reliability evaluation, are formulated and proved. The classes of limit reliability functions for homogeneous and non-homogeneous series, parallel, series-parallel and parallel-series multi-state systems and for a homogeneous multistate "m out of n" system are established. Practical applications of the multi-state asymptotic approach to reliability evaluation of real technical systems are presented.

On the basis of auxiliary theorems some corollaries are formulated and proved and then applied to approximate reliability and risk characteristics determination of real technical multi-state systems having series, parallel, "m out of n", series-parallel and parallel-series reliability structures. Evaluations are given of multi-state reliability functions, mean sojourn times in the state subsets and their standard deviations, mean lifetimes in the states, risk functions, and exceeding moments of a permitted risk level for selected real systems.

The homogeneous series piping transportation system, the model homogeneous series telecommunication network, the homogeneous series bus transportation system, the non-homogeneous series piping transportation system, the homogeneous parallel system of an electrical cable, the non-homogeneous parallel rope system, the "10 out of 36" homogeneous steel rope system, the model homogeneous series-parallel system, the homogeneous and non-homogeneous series-parallel pipeline systems and the homogeneous parallel-series electrical energy distribution system are analysed and their reliability characteristics are evaluated.

Necessary data on system components reliability and system operation processes come from experts, from trade standards and from certificates issued by the system component producers. Component reliability and system operation processes data are, out of necessity, approximate and they concern the components' mean lifetimes in the reliability state subsets and the hypothetical distributions of these lifetimes. The accuracy of the asymptotic approach to the reliability evaluation of the considered systems is illustrated in tables and figures.

6. Reliability evaluation of port and shipyard transportation systems

The multi-state asymptotic approach is applied to the reliability and risk characteristics evaluation of selected large transportation systems used in ports and shipyards. Reliability analysis of multi-state series, series-parallel and parallel-series transportation systems is based on corollaries formulated and proved in this and the preceding chapters. The corollaries are applied to evaluate reliability characteristics of three transportation systems used at the Port of Gdynia and one operating at the Naval Shipyard of Gdynia.

The port grain transportation system built of three-state non-homogeneous series-parallel subsystems, the port oil piping transportation system composed of three-state non-homogeneous series-parallel subsystems, the port bulk transportation system built of four-state non-homogeneous series-parallel and series subsystems and the shipyard rope transportation system that is a four-state homogeneous parallel-series system, are considered.

Multi-state reliability functions, mean values of sojourn times in the state subsets and their standard deviations, mean values of lifetimes in the particular states, risk functions, and exceeding moments of a permitted risk level are determined for these systems.

The accuracy of the asymptotic approach to the reliability evaluation of these systems is also illustrated. System components reliability data and system operation processes data come from operators of these systems, component technical certificates and binding standards. Reliability data are, out of necessity, approximate and they concern only the mean values of the system components' sojourn times in the state subsets and hypothetical distributions of these lifetimes.

7. Reliability of large multi-state exponential systems

Limit reliability functions of multi-state series, parallel, "m out of n", series-parallel and parallel-series systems composed of components having exponential reliability functions, are established. Next, the results are presented in the form of tables containing exact algorithms of the procedure while evaluating reliability characteristics of these systems' reliability in order to deliver a simple and convenient tool for everyday practice of the reliability practitioners. The tables are composed of three parts, containing reliability data of the evaluated system, necessary calculations, and results of the system reliability evaluation. The way of using the algorithms is illustrated by several examples.

8. Related and open problems

Domains of attraction for limit reliability functions of two-state systems are introduced. They are understood as the conditions which the reliability functions of the particular components of the system have to satisfy in order the system limit reliability function to be one of the limit reliability functions from the previously established class for this system. Exemplary theorems concerning domains of attraction for limit reliability functions of homogeneous series systems are presented and the application of one of them is illustrated.

A practically important problem of accuracy of the asymptotic approach to large systems reliability evaluation, which concerns the speed of convergence of system reliability sequence, is discussed. This problem is illustrated by analysing the speed of convergence of the homogeneous series-parallel system reliability sequences, to its limit reliability function. Series - -"m out of n" systems and "m out of n"- series systems are defined and exemplary theorems on their limit reliability functions are presented and applied to the reliability evaluation of an illumination system and a rope elevator.

Hierarchical series-parallel and parallel-series systems of any order are defined, their reliability functions are determined and limit theorems on their reliability functions are applied to reliability evaluation of exemplary hierarchical systems of 2nd order.

Applications of the asymptotic approach to large series systems reliability improvement are also presented. The chapter is completed by showing the possibility of applying the asymptotic approach to the reliability analysis of large systems related to their operation processes. In this scope, the asymptotic approach to reliability evaluation for a large port grain transportation system related to its operation process is performed.

Miscellanea

ZTM next tenure

On 20 May 2004 Marine Technology Unit (acting within the Transport Technical Means Section , Transport Committee, Polish Academy of Sciences) commenced its next tenure of activity. ZTM's meeting was held at the Faculty of Ocean Engineering and Ship Technology, Gdańsk University of Technology, and chaired, as usually, by Prof. J. Girtler.

The new tenure was inaugurated
by presenting two scientific papers :

- *Possible applications of semi-Markovian reliability models to ship power plant operation processes* – by J. Rudnicki D.Sc.
- *Probabilistic models in designing the power systems of bucket dregers* – by D. Bocheński D.Sc.

Both authors are scientific workers of the Faculty.

After discussion on the presented topics the second, organizational part of the meeting took place. The last-year activity of the Unit was summed up and evaluated, the activity program for the year 2005 presented, as well as several other organizational issues discussed.