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POLISH MARITIME RESEARCH is a scientific journal of worldwide circulation. The journal appears as a quarterly four times a year. The first issue of it was published in September 1994. Its main aim is to present original, innovative scientific ideas and Research & Development achievements in the field of:

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which could find applications in the broad domain of maritime economy. Hence there are published papers which concern methods of the designing, manufacturing and operating processes of such technical objects and devices as: ships, port equipment, ocean engineering units, underwater vehicles and equipment as well as harbour facilities, with accounting for marine environment protection.

The Editors of POLISH MARITIME RESEARCH make also efforts to present problems dealing with education of engineers and scientific and teaching personnel. As a rule, the basic papers are supplemented by information on conferences, important scientific events as well as cooperation in carrying out international scientific research projects.

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Semi-Markovian models of the process of technical state changes of technical objects

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ABSTRACT



The most important problem of operation of all technical objects (devices) is the problem of rational (optimum) decision control of the process of technical state changes of the objects. Such control can be realized when a model of the process of technical state changes is applied. In view of this, a formal description of the operational process of technical objects was presented in this paper, as well as it was justified that the model of the process may be a semi-Markovian process of finite set of states. Examples of 4-and 3-state semi-Markovian process were given as a model of the above mentioned process of state chan-

ges of any device. Difficulties associated with forming such model, which arise from application of the theory of semi-Markovian processes, were indicated. Therefore the proposed model is the semi-Markovian process whose values are technical states occurring in the phase of operation of many technical objects of practical importance (e.g. combustion piston engines, turbines, displacement and rotary compressors, impeller pumps etc).

Key words: model, technical object, semi-Markovian process, technical state

INTRODUCTION

The process of technical state changes of any technical object is the process of occurence of successive technical states casually linked during time t, which appear one by one in such a way that the successive state occurs after a determined duration time interval of the directly preceding state. The duration time intervals of particular states are random variables of continuous realizations and finite expected values.

The process of technical state changes of every technical object belongs to the most important processes occurring in the phase of its operation. Course of the process should be rational, i.e. that whose realization results from an assumed optimization criterion. Such criterion may be e.g. expected value of operational cost of a given technical object or instantaneous availablity factor. The last criterion is important when in any instant of operation of particular technical objects, t, a task can be assigned to them for realization of which their reliable operation is necessary [7, 8]. The control which makes the optimum course of the process of occurrence of successive technical states possible, can be realized when the model of the process, which allows for application one of the decision theories, is elaborated. In the case of such technical objects as combustion piston engines and turbines, positive-displacement and axial-flow compressors two theories are of importance: the statistical decision theory and the theory of controlled semi--Markovian processes [8, 10, 11, 12, 13]. Recently the theory of controlled semi-Markovian processes is more and more often applied with success to solving different problems of durability, reliability and decision-based operational control of various technical objects. The theory may also find use in solving similar problems associated with operation of many technical objects including those dealing with control of the process of technical state changes of the objects. For this reason the semi-Markovian model of the process has been proposed and its practical applicability justified.

PREMISES FOR ELABORATION OF SEMI--MARKOVIAN MODEL OF TECHNICAL STATE CHANGES OF OBJECTS

From the definition of semi-Markovian process it results [12, 17, 20, 21] that such process is stochastic and of a discrete set of states, and its realizations are functions of constant intervals (i.e. those having uniform values within operation time intervals being random variables), and right-hand continuous ones. From the definition it also results that the process is determined when its initial distribution $P_i = P\{Y(0) = s_i\}$ as well as its matrix function $Q(t) = [Q_{ij}]$ whose elements are probabilities of transfer of the process from the state s_i to the state s_j , during the time not greater than t ($i \neq j$; i,j = 1, 2, ..., k), are known. The elements $Q_{ij}[(i \neq j; i,j = 1, 2, ..., k)]$ of the matrix Q(t) are non-decreasing functions of the variable t [12, 17]. Non-zero elements of the matrix can be interpreted as follows:

$$\begin{split} Q_{ij}(t) &= P\left\{W(\tau_{n+1}) = s_j \ ; \ \tau_{n+1} - \tau_n \le t \ \middle| \ W(\tau_n) = s_i \right\} \ ; \\ s_i \ , \ s_i \in S \ ; \ i \ , j = 1, \, 2, \, 3, \, 4 \ ; \ i \ne j \end{split} \tag{1}$$

The semi-Markovian model of real process of occurrence of successive technical states of any technical object can be formed only when determination of states of the process is possible in such a way as to obtain duration time of the existing state in the instant τ_n as well as the state available in the instant τ_{n+1} , stochastically independent on the preceding states and their duration times [6 , 12]. Hence in the modelling which is supposed to lead to elaboration of a semi-Markovian model of the technical state changing process of given technical objects the analysis of changes of technical states occurring in real operational conditions of the objects should be accounted for.

In the case of such technical objects as self-ignition engines, piston compressors, impeller pumps etc it has been observed that prediction of technical state changes of the objects can be performed if actual technical state and future conditions of their operation are known [5, 6, 7]. This fact which simultaneously means that technical state changes of such objects are not strictly dependent on their time of operation, can be highlighted by means of the following hypothesis **H1**:

the technical state changing process of any technical object (understood as a random time function whose values are random variables representing existing technical states), which occurs in a rational operation system (i.e. in such operation system where operational cost calculations are carried out), is the process of asymptotically independent values because its arbitrary state considered in any instant $\tau_n\ (n=0\ ,1\ ,...,m\ ;$ $\tau_0<\tau_1<...<\tau_m)$ significantly depends on the state directly preceding it, but not on the states which have occurred earlier and their duration time intervals.

It should be observed that the hypothesis does not contain any contradictions which could falsify it in a logical sense before testing it.

The consequences of the hypothesis are as follows [5, 7]:

- **⊃** the probabilities $(p_{ij}; i \neq j; i, j \in N)$ of transfer of the state changing process of any technical object from any actual state of the object, s_i , to any next state s_j do not depend on the states in which the process has been before
- lacktriangle the unconditional duration time intervals of the particular states s_i of the technical state changing process of a technical object are the random stochastically independent variables $(T_i ; i \in N)$
- **⊃** the duration time intervals of any possible state s_i of the technical state changing process of a technical object, provided that the next state will be one of the remaining states s_j of the process, are the random stochastically independent variables $(T_{ij}; i \neq j; i,j \in N)$.

The above mentioned consequences reveal the probabilistic law of changing the technical states of any technical object. They are not contradictory to each other, and their logical veracity is doubtless. Therefore there is no obstacle to consider the consequences as one common consequence K1, in order to use it for empirical verification of the formulated hypothesis H1. Such verification consists in experimental investigation of veracity of the enumerated consequences taken as one common consequence K1.

The hypothesis can be supported by the more detailed hypothesis H2 dealing with all technical objects fitted with tribological units (e.g. self-ignition engines, piston compressors etc). The process of changing their technical states is that in which the duration time intervals of each of the states are random variables. Particular realizations of the random variables depend on many factors, a.o. on wear quantity of the tribological units of the objects. For the objects in question (e.g. self-igni-

tion engines) it was observed that wear of their sliding tribological units is weakly correlated with time [3, 4, 9, 14, 18, 26]. The observation is important because serviceability of such objects mainly depend on technical state (i.e. wear) of their tribological units. This made it possible to predict technical state of such objects by taking into account only their current state as well as service conditions without accounting for the states occurred before. In order to highlight the fact the following hypothesis **H2** can be given:

the state of any sliding tribological unit of any technical object, as well as its duration time significantly depend on the preceding state but not on those occurred before and on their duration time intervals because its load and both wear rate and wear increments induced by the load are the processes of asymptotically independent values.

The statement contained in the hypothesis results from two obvious facts:

- ★ there is a close relationship between the loading of tribological units of different technical objects (e.g. self-ignition engines, piston compressors etc) and their wear [16, 18, 26, 27]
- ★ in a longer operation period of any technical object (e.g. self-ignition engines, piston compressors etc) there is no monotically increasing load changes of their tribological units, hence the service loading of the units can be assumed stationary [2, 4, 15, 22, 23, 24, 25].

In order to verify the presented hypothesis H2 and to determine if it is true, it is necessary to predict the consequences whose occurrence is possible to be empirically checked.

The consequences K2 which can be concluded from the hypothesis (with accounting for loading features of the objects and their sliding tribological units) were presented in [6].

Verification of the presented hypothesis H1 and H2 by means of experimental testing the veracity of the consequences K1 and K2 can be performed with the use of the same reasoning methods which were presented in [3, 4, 7].

VARIANTS OF SEMI-MARKOVIAN MODEL OF TECHNICAL STATE CHANGES OF OBJECTS

From the presented hypotheses H1 and H2 it results that models of the technical state changing process $\{W(t): t \ge 0\}$ of many technical objects such as e.g. self-ignition engines can be the stochastical processes of discrete set of states and continuous time being the duration time of distinguished technical states of the objects. The considered models of the technical state changing process of any technical object can be mathematically expressed by the functions mapping the set of instants, $T \in R_+$, into the set of technical states, S. Hence to elaborate such model it is necessary to determine a finite set of technical state changes of the objects in question. Assuming the serviceability of technical objects as the criterion for distinguishing the states one can distinguish the set of classes (subsets) of technical states (shortly called "states"), S. The below given set can be deemed a set of the states of practical operational importance [10]:

$$S = \{s_i : i = 1, 2, 3, 4\}$$
 (2)

which have the following interpretation:

♦ s₁ – the state of full serviceability, i.e. the technical state of any technical object, which makes it possible to use the object within the whole range of loads to which it was adjusted in the phases of its designing and manufacturing

- ♦ s₂ the state of partial serviceability, i.e. the technical state of any technical object, which makes it possible to fulfil all its tasks (as in s₁) but at lower values of operational indices (e.g. at a lower overall efficiency, hence in the case of self-ignition engine at a greater specific fuel oil consumption)
- ♦ s₃ the state of limited unserviceability, i.e. the technical state of any technical object, which makes it possible to fulfil only some of its tasks (e.g. such state which in the case of self-ignition engine precludes it from working in accordance with maximum continuous rating characteristics, and in the case of piston compressor precludes it from filling the air receiver up to a required air pressure, etc)
- ♦ s₄ the state of full unserviceability, i.e. the technical state of any technical object, which precludes the object from fulfilling any task of the set of the tasks to which it was adjusted in the phases of its designing and manufacturing (for instance such state of engine, which precludes it from work in accordance with maximum continuous rating characteristics.

Elements of the set $S = \{s_i \; ; \; i=1,2,3,4\}$ are values of the process $\{W(t): t \geq 0\}$ composed of the states successively occurring one by one, $s_i \in S$, and being casually related to each other.

In the case of many technical objects (such as self-ignition combustion engines, gas turbines, piston compressors, etc.) the distinguishing of the states $s_i \in S$ (i = 1, 2, 3, 4) is so much important because their use is crucial when they are in the state s_1 or in the state s_2 . However in the second case the objects should be used for as-short-as-possible period only after which they should be renewed to bring them back to the state s_1 . The states are values of the process $W(t): t \ge 0$ which is fully determined if its functional matrix is known [9, 12]:

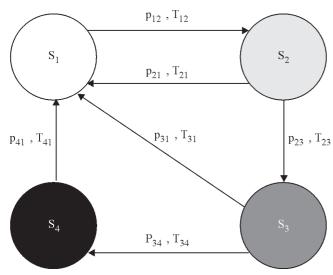
$$\mathbf{Q}(t) = [Q_{ii}(t)] \tag{3}$$

as well as its initial distribution is given:

$$p_i = P\{W(0) = s_i\}$$
 $s_i \in S$; $i = 1, 2, 3, 4$ (4)

Depending on an assumed strategy of maintaining the technical objects in the technical states making it possible to realize their tasks, different realization variants of the process $\{W(t):t\geq 0\}$ may be taken into account.

In the first variant a technical state change can occur in compliance with the graph of technical state changes presented in the Figure.



Graph of changes of the technical states $s_i \in S(i = 1, 2, 3, 4)$ of the process $\{W(t): t \ge 0\}$

The following initial distribution of the process $\{W(t): t \ge 0\}$ can be assumed:

$$p_1 = P\{W(0) = s_1\} = 1$$

$$p_i = P\{W(0) = s_i\} = 0 \text{ for } i = 2, 3, 4$$
(5)

and its functional matrix in the following form complying with the graph of Figure :

$$\mathbf{Q}(t) = \begin{bmatrix} 0 & Q_{12}(t) & 0 & 0 \\ Q_{21}(t) & 0 & Q_{23}(t) & 0 \\ Q_{31}(t) & 0 & 0 & Q_{34}(t) \\ Q_{41}(t) & 0 & 0 & 0 \end{bmatrix}$$
(6)

The matrix (6) represents changes of the states $s_i \in S(i=1,2,3,4)$ of the process $\{W(t):t\geq 0\}$. The probabilities $P_j(j=1,2,3,4)$ of the event that a given technical object will be in the states $s_i \in S(i=1,2,3,4)$ determine a chance of fulfilling the task by the object. It is obvious that the user of every technical object is interested in the object to be in the state s_1 as long as possible. The chance of lasting any technical object in this state is determined by the probability determined for an appropriately long time of operation. It means that the limiting distribution of the process should be determined.

From the semi-Markovian process theory it results [12, 17, 21] that the probabilities of changes of states of any technical object are determined by the probabilities p_{ij} of the Markov's chain $\{W(\tau_n): n=0,1,2,...\}$ introduced into the process $\{W(t): t \geq 0\}$. The probabilities form the following matrix of transfer probabilities:

$$P = [p_{ij} ; i,j = 1, 2, 3, 4]$$
 where:

$$p_{ij} = P\{W(\tau_{n+1}) = s_j \mid W(\tau_n) = s_i\} = \lim_{t \to \infty} Q_{ij}(t)$$

The matrix (7) makes determining the limiting distribution of the process $\{W(t): t \ge 0 \text{ possible. From the matrix (6) it results that the matrix (7) has the following form :$

$$\mathbf{P} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \mathbf{p}_{21} & 0 & \mathbf{p}_{23} & 0 \\ \mathbf{p}_{31} & 0 & 0 & \mathbf{p}_{34} \\ 1 & 0 & 0 & 0 \end{bmatrix}$$
(8)

From the theorem given in [12] it results that the limiting distribution of the considered process exists:

$$\begin{aligned} P_{j} &= \lim_{t \to \infty} P\{W(t) = s_{j}\} = \\ &= \lim P\{W(t) = s_{j} / W(0) = s_{j}\} \end{aligned} \tag{9}$$

which is determined by the expression:

$$P_{j} = \frac{\pi_{j}E(T_{j})}{\sum_{k=1}^{4} \pi_{k}E(T_{k})} \quad ; \quad j = 1, 2, 3, 4$$
 (10)

and the limiting distribution π_j (j=1,2,3,4) of the introduced Markov's chain $\{W(\tau_n): n=0,1,2,...\}$ fulfils the following equations:

$$\begin{bmatrix} \pi_1, \pi_2, \pi_3, \pi_4 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ p_{21} & 0 & p_{23} & 0 \\ p_{31} & 0 & 0 & p_{34} \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \pi_1, \pi_2, \pi_3, \pi_4 \end{bmatrix}$$
 (11)

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1 \tag{12}$$

Making use of the relationships $(10) \div (12)$ one obtains the following formulae :

$$P_1 = E(T_1)M^{-1}$$
; $P_2 = E(T_2)M^{-1}$
 $P_3 = p_{23}E(T_3)M^{-1}$; $P_4 = p_{23}p_{34}E(T_4)M^{-1}$ (13)
and:

$$M = E(T_1) + E(T_2) + p_{23}E(T_3) + p_{23}p_{34}E(T_4)$$
 where :

 $E(T_j)$ - expected value of duration time of the state $s_i \in S$ (j = 1, 2, 3, 4)

 $\begin{array}{ll} p_{ij} & \text{-probability of transfer of the process} \\ \{W(t): t \geq 0\} \text{ from the state } s_i \text{ to the state } s_j \\ (s_i \,, s_j \in S \;\; ; \;\; i,j = 1, \, 2, \, 3, \, 4 \; ; \; i \neq j). \end{array}$

The particular probabilities $P_j(j = 1, 2, 3, 4)$ given by the formulae (13) have the following interpretation:

$$\begin{split} P_1 &= \lim_{t \to \infty} \ P\{W(t) = s_1\} \quad ; \quad P_2 = \lim_{t \to \infty} \ P\{W(t) = s_2\} \\ P_3 &= \lim_{t \to \infty} \ P\{W(t) = s_3\} \quad ; \quad P_4 = \lim_{t \to \infty} \ P\{W(t) = s_4\} \end{split}$$

In the presented variant, such situations are accounted for in which the user is able to risk to fulfil his task in the state s_2 of the technical object and even to risk to fulfil some tasks in the state s_3 of the object.

The second variant deals with the case when, within an assumed operational strategy of technical objects, the distinguishing between the states s_1 and s_2 is of no importance. Then it is possible to consider the simpler process $W(t):t\geq 0\}$ of technical state changes of the objects, namely the model having the set of states :

$$S = \{s_1, s_2, s_3\} \tag{14}$$

as well as their following interpretation [6, 7, 8]:

- ★ the state of full serviceability, s₁, which makes it possible to use the object in any conditions and in any range of loads, to which it was adjusted in the phases of its designing and manufacturing
- ★ the state of partial serviceability, s₂, which makes it possible to use the object in limited conditions and in a range of loads lower than those to which it was adjusted in the phases of its designing and manufacturing
- ★ the state of unserviceability, s₃, which does not make it possible to use the object in accordance with the purpose it was intended for (e.g due to its failure, performing maintenance operations on its subassemblies etc).

A graph of technical state changes of such process W(t): $t \ge 0$ } of objects of the kind as well as its example course were presented in [6]. Hence the process is the three-state one of continuous realizations (i.e. time-continuous). It may be assumed that if anyone of the states s_2 or s_3 does not occur then every technical object remains in the state s_1 . And, the set of the technical states $S = \{s_1, s_2, s_3\}$ can be considered as the set of values of the stochastic process $\{W(t): t \ge 0\}$ of realizations constant within intervals, and being right-hand continuous.

The initial distribution of the considered process having transfer graph [6, 8] is given by the formula:

$$P_{i} = P\{W(0) = s_{i}\} = \begin{cases} 1 & \text{for } i = 1 \\ 0 & \text{for } i = 2,3 \end{cases}$$
 (15)

and its functional matrix, if the function $Q_{32}(t)$ is different from zero $(Q_{32}(t) \neq 0)$, is the following:

$$\mathbf{Q}(t) = \begin{bmatrix} 0 & Q_{12}(t) & Q_{13}(t) \\ Q_{21}(t) & 0 & Q_{23}(t) \\ Q_{31}(t) & Q_{32}(t) & 0 \end{bmatrix}$$
(16)

In operational practice of technical objects any partial renewal strategy of their technical states should not be applied. Use of such strategy may lead to occurrence of the unserviceability state s_3 and, as a result, - e.g. to occurrence of a high economical loss. It means that renewals can be applied in special cases only, e.g. those resulting from impossibility to discontinue realization of the undertaken task and to complete the technical state renewal of the object. Therefore operation process of every technical object may be rational when the function $Q_{32}(t) = 0.$ In this case the matrix (16) obtains the form :

$$\mathbf{Q}(t) = \begin{bmatrix} 0 & Q_{12}(t) & Q_{13}(t) \\ Q_{21}(t) & 0 & Q_{23}(t) \\ Q_{31}(t) & 0 & 0 \end{bmatrix}$$
(17)

And, like in the case of the earlier considered processes, for the presented process $\{W(t):t\geq 0\}$ having the functional matrix given by (17), the following limiting distribution can be determined:

$$P_{1} = \frac{E(T_{1})}{H} ; P_{2} = \frac{p_{12}E(T_{2})}{H}$$

$$P_{3} = \frac{(1 - p_{12}p_{21})(E(T_{3})}{H}$$
(18)

$$H = E(T_1) + p_{12}E(T_2) + (1 - p_{12}p_{21})E(T_3)$$

where:

 P_1, P_2, P_3 - probabilities of the event that a given technical object is in the state : s_1, s_2, s_3 , respectively

 p_{ij} - probability of transfer of the process $\{W(t): t \ge 0\}$ from the state s_i to the state s_i

 $E(T_i)$ - expected value of duration time of the state s_i .

The particular probabilities $P_j(j=1,2,3,4)$ given by (18) can be interpreted as follows:

$$\begin{split} P_1 &= \lim_{t \to \infty} \ P\{W(t) = s_1\} \quad ; \quad P_2 = \lim_{t \to \infty} \ P\{W(t) = s_2\} \\ P_3 &= \lim_{t \to \infty} \ P\{W(t) = s_3\} \end{split}$$

The presented probabilities which determine possible occurrence of particular states of any technical object, are essential in making operational decisions [1, 7, 8, 10, 13].

SUMMARY

- ➤ In the presented considerations it is shown that the process of technical state changes of different technical objects (self-ignition engines, gas turbines, piston compressors, impeller pumps etc) is a process continuous over time and states. Because technical state of every technical object is subject to continuous changes hence it is possible to consider the sets composed of infinite number of technical states.
- Identification of all technical states of any technical object is neither possible nor purposeful for both technical and economical reasons. Therefore a need arises to split the set of technical states into a finite number of classes (subsets) of technical states.

- Assuming the serviceability of technical objects to be a splitting criterion one can distinguish (in the simplest case) the following classes of of their technical states: the full serviceability state s₁, the partial serviceability state s₂, and the unserviceability state s₃.
- ➤ The set of the states $S = \{s_1, s_2, s_3\}$ may be considered as the set of values of the simplest stochastic process $\{W(t): t \geq 0\}$ whose realizations are constant within intervals and right-hand continuous. The process, a model of real process of technical state changes of objects, is mathematically described by a function mapping the set of the instants T ($T \in R_+$) into the set of technical states S. Elaboration of such a model adequate to the process of technical state changes of the objects is indispensable for rational control of the process.
- The model presented in the form of the stochastic process {W(t): t≥0} is the simplest and fulfilling the two conditions:
 - it works like the original, i.e. it realizes analogical functions
 - it makes it possible to reveal on the basis of analysis of
 its structure and functioning mode the new, hidden features of the processes of technical state changes of the
 objects in question, which are represented by this model,
 namely: the reliability indices expressed by the probabilities, P_i, of lasting the process in distinguished states.
- ➤ From the presented hypotheses it results that the process of technical state changes of the objects in question can be investigated by means of the models formed as semi-Markovian processes. Therefore the process {W(t): t ≥ 0} can be considered as a semi-Markovian process of real processes of technical state changes of many technical objects.
- ➤ The hypotheses should be verified for each kind of technical objects, e.g. self-ignition engines, piston compressors, impeller pumps etc, as it may happen that for a given kind of technical objects operating in assumed service conditions it would not be possible to use the models of technical state changes in the form of semi-Markovian processes.

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Miscellanea

Flutter of Turbine Rotor Blades in Inviscid Flow

In the year 2004 Polish Naval University in Gdynia edited (in English) the book on: "Flutter of Turbine Rotor Blades in Inviscid Flow"

written by Prof. Romuald Rzadkowski

The motivation to write this monography was to present theoretical models, numerical codes and results of the aero-elastic calculations of rotor blades which have been done since 1997 in the Institute of Fluid-Flow Machinery, Polish Academy of Sciences in Gdańsk.

These results were obtained in collaboration with Prof. V. Gnesin from Institute for Machinery Problems, Ukrainian National Academy of Sciences, Kharkov branch, and dr V. Tsimbalyuk from the Institute for Strength Problems, Ukrainian National Academy of Sciences, Kiev.

Turbine blading dynamics is an interdisciplinary domain of knowledge, which inter-connects mechanics of materials, vibration theory, aerodynamics, thermal exchange, problems of corrosion and erosion effects of surrounding medium, and manufacturing technology. The most important information required to perform the dynamic calculations is the blade unsteady force which arises out of the flow path interaction in the stage and the flutter parameters.

The presented book is limited to the flutter of rotor blades in inviscid flow.

It includes physical and mathematical modelling of self-excited vibration in 2D and 3D flows. For building mathematical models of blade and bladed disc motion the general theory of continuous media and methods of analytical mechanics were employed.

The entire scope of the book is contained in the seven chapters.

- O Chapter 1 is introductory
- O In **Chapter 2** the flutter phenomena and aeroelasticity methods are described.
- O Chapter 3 presents the 1D and 3D structure models of a blade and a bladed disc.
- O In Chapter 4 the 2D inviscid flutter model is given. The 2D trans-sonic flow of an ideal gas through a multipassage blade row is considered. In a general case the flow is assumed to be an aperiodic function from blade to blade (in the pitchwise direction), so the calculated domain includes all blades of the whole assembly. The aerodynamic model fully accounts for the blade thickness, camber and the angle-of-attack effects. The unsteady flow of the ideal gas is described by the 2D Euler equations. Godunov-Kolgan method is used to descritize the 2D Euler equations. The numerical test calculations were performed to compare the theoretical results obtained by using mathematical model developed here, with experimental data.
- O In Chapter 5 the model of 3D self-excited vibration of bladed disc in the compressible flow is shown. The 3D non-linear time-marching method for aeroelastic beha-

viour of oscillating turbine blade row was presented. The numerical calculations were performed to compare the theoretical results of 3D inviscid flutter code with experiments. The aeroelastic behaviour of a 3D oscillating row of last stage turbine blades of L=0.765 [m] in length is shown for a harmonic motion and coupled fluid-structure interaction.

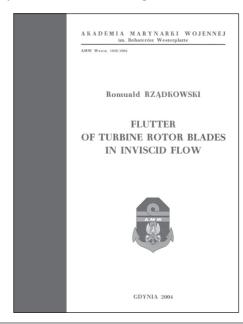
- O In **Chapter 6** the experimental stand to simultaneously measure unsteady aerodynamic force and moment with arbitrary combinations of blade motions in the subsonic flow, is presented. The numerical calculations were performed to compare the theoretical results with experiments for the harmonic motion.
- O Chapter 7 presents the general conclusions and remarks and the directions of the future work.

The book is addressed to the scientific workers and students of the university faculties engaged in the field of aeroelasticity applied to steam and gas turbines.

The presented book was preceded by another interesting book of the same author, titled: **Dynamics of Rotor Steam Turbine Blading**, Part Two, **Bladed Discs** edited by Maszyny Przepływowe, Wrocław, Ossolineum, 1998.

Prof. Romuald Rządkowski graduated from Shipbuilding Institute, Gdańsk Technical University, and Faculty of Mathematics, Gdańsk University. Presently he works in the Institute of Fluid-Flow Machinery, Polish Academy of Sciences, and Polish Naval Academy in Gdynia. He obtained his PhD degree in 1988 and DSc in 1998 at the Institute of Fluid-Flow Machinery, Polish Academy of Sciences.

Prof. Romuald Rządkowski is the author and co-author of about 123 scientific papers. The area of his scientific interest covers free and forced vibration of turbine blades, bladed discs, shaft with bladed discs, life estimation, flutter and unsteady forces in the turbine stages.



Viscoelastic lubrication of spherical slide bearing in impulsive unsteady motion

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ABSTRACT



This paper presents numerical and semi-analytical solutions of oil velocity components and pressure distributions in spherical unsymmetrical gap of slide bearing. A hydrodynamic unsteady lubrication during oil flow with viscoelastic properties is here considered. In the case of various driving systems on ships the bearings with spherical journals and spherical sleeves or slide bearings with spherical bits operate often under impulsive unsteady motions. Many impurities appearing in service leads to viscoelastic properties of the oil. During service of transport machines it is necessary to adjust the shaft location respective

to the sleeve in order to make optimizing the convergent lubricating film possible. Such conditions are effectively satisfied in bearings with spherical journals. The presented numerical calculations were performed by means of the Mathcad 2000 Professional Program and the method of finite differences. This method satisfies stability conditions of numerical solutions of capacity forces occurring in spherical bearings.

Key words: driving systems on ships, spherical slide bearing, unsteady impulsive viscoelastic lubrication

INTRODUCTION

Lubrication of spherical bearing under periodic motion has been considered in many papers till now [3, 4, 8, 9, 10, 11, 12]. This paper considers pressure distribution during hydrodynamic viscoelastic lubrication of spherical bearings and bearings with spherical bit, at impulsive unsteady motion. These problems have been not elaborated hitherto.

Bearing systems are commonly used in diesel engines installed in land transport machines and ships. The oil in bearing gaps in such bearing systems is contaminated mainly with dust, soot, smoke black as well as many inhibitors improving the oil properties. Transport machines usually work under unsteady impulsive vibrations. Thus the lubricating oil has often the non-Newtonian properties. Therefore in this paper viscoelastic time-dependent properties of oil are taken into account. Designing the bearings without accounting for the viscoelastic oil properties brings about to occurrence of the seizing of the bearing in kinematics pairs.

The seizing of bearings can be prevented by proper recognition of bearing operational parameters for real oils. This is very important because the seizing of ship diesel engine bearing system may lead to the catastrophe [1, 2]. Therefore determination of real bearing capacity at unsteady viscoelastic lubrication has important sense.

The bearings presented in this paper have spherical journals. The spherical journals can be turned with the shaft or the form of an individual ball can be used (see Fig.1). Such ball is installed in bored end of the shaft.

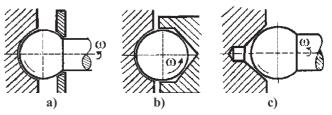


Fig.1. Spherical bearings: a) spherical journal together with shaft and spherical sleeve, b) individual spherical ball with conical sleeve, c) spherical journal together with shaft and conical sleeve

The sleeve has spherical or conical shape. The spherical sleeve is more effective than conical one, because it ensures small values of slide thrust and wear.

Usually the spherical bearing is adjustable. It makes it possible to set up the shaft with respect to the sleeve and to control the convergent lubricating film, and it is capable of transferring both axial and transverse loading, Fig.2.

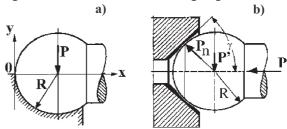


Fig. 2. Loading of the spherical journal: a) transverse loading, b) axial and transverse loading

GOVERNING EQUATIONS AND BOUNDARY LAYER SIMPLIFICATIONS

Lubrication of journal and sleeve in slide spherical bearing is described by oil flow. The momentum conservation equations and continuity equation describe oil flow. Moreover the second order approximation of the general constitutive equation given by Rivlin and Ericksen, can be considered. The equations can be expressed in the following form [6, 7]:

Div
$$\mathbf{S} = \rho d\mathbf{v}/dt$$
, div $\mathbf{v} = 0$, $\mathbf{S} = -p\mathbf{I} + \eta_0 \mathbf{A}_1 + \alpha (\mathbf{A}_1)^2 + \beta \mathbf{A}_2$ (1)

oil density

 A_1 and A_2 - two Rivlin-Ericksen strain tensors of three material constants η_o, α, β ,

 η_0 - dynamic viscosity α , β - pseudo-viscosity constans of oil.

The tensors A_1 , and A_2 are given by the symmetric matrices defined by:

$$\mathbf{A}_{1} \equiv \mathbf{L} + \mathbf{L}^{T}$$

$$\mathbf{A}_{2} \equiv \operatorname{grad} \mathbf{a} + (\operatorname{grad} \mathbf{a})^{T} + 2\mathbf{L}^{T}\mathbf{L}$$

$$\mathbf{a} \equiv \mathbf{L}\mathbf{v} + \frac{\partial \mathbf{v}}{\partial \mathbf{t}}$$
where:

 $\begin{array}{ll} \mathbf{L} & \text{- tensor of oil velocity vector gradient } [s^{\text{-}1}] \\ \mathbf{L}^T & \text{- tensor with matrix transpose } [s^{\text{-}1}] \end{array}$ a - acceleration vector [m/s²]
 grad a - acceleration vector gradient

It is assumed that the product of Deborah and Strouhal numbers, i.e. DeStr, and the product of Reynolds number, dimensionless radial clearance, and Strouhal number, i.e. Re ψ Str, are of values of the same order. Moreover DeStr >> De $\equiv \alpha\omega/\eta_0$. where : ψ - relative radial clearance ω - angular velocity of spherical bearing journal.

The following is additionally assumed:

- > the rotational motion of spherical journal with peripheral tangential velocity $U = \omega R$
- unsymmetrical unsteady oil flow in the gap
- viscoelastic and unsteady properties of oil
- \triangleright the oil density ρ of constant value
- \triangleright the characteristic value of the bearing gap height, ϵ
- > no slip at the bearing surfaces
- R radius of spherical journal.

By neglecting the terms of the radial clearance $\psi \equiv \epsilon/R \cong 10^{-3}$ in the governing equations expressed in the spherical coordinates ϕ , r, ϑ , and by taking into account the above mentioned assumptions the following is obtained:

$$\frac{\partial v_{\phi}}{\partial t} = -\frac{1}{\rho R \sin \frac{\theta}{R}} \frac{\partial p}{\partial \phi} + \frac{\eta_{o}}{\rho} \frac{\partial}{\partial r} \left(\frac{\partial v_{\phi}}{\partial r} \right) + \frac{\beta}{\rho} \frac{\partial^{3} v_{\phi}}{\partial t \partial r^{2}}$$
(3)

$$0 = \frac{\partial \mathbf{p}}{\partial \mathbf{r}} \tag{4}$$

$$\frac{\partial v_9}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial 9} + \frac{\eta_o}{\rho} \frac{\partial}{\partial r} \left(\frac{\partial v_9}{\partial r} \right) + \frac{\beta}{\rho} \frac{\partial^3 v_9}{\partial t \partial r^2}$$
 (5)

$$\frac{\partial v_{\varphi}}{\partial \varphi} + R \sin\left(\frac{\vartheta}{R}\right) \frac{\partial v_{r}}{\partial r} + \frac{\partial}{\partial \vartheta} \left[Rv_{\vartheta} \sin\left(\frac{\vartheta}{R}\right) \right] = 0 \tag{6}$$

where : $0 \le \phi \le 2\pi c_1$, $0 < c_1 < 1$, $b_m \equiv \pi R/8 \le \vartheta \le \pi R/2 \equiv b_s$, $0 \le r \le h$, h-gap height. Symbols v_ϕ , v_r , v_ϑ denote oil velocity components in the circumferential, gap-height and meridianal directions of the spherosphere. rical journal, respectively. The terms multiplied by the coefficient β in the right hand sides of (3), (5) denote influence of timevariable viscoelastic oil properties on the bearing operational parameters. The terms in the left hand sides of (3), (5) describe influence of accelerations which occur during the impulsive motion, on the bearing lubrication. The relationships between dimensional and dimensionless quantities are assumed in the following form:

$$r = \varepsilon r_1$$
, $\vartheta = R\vartheta_1$, $t = t_o t_1$, $h = \varepsilon h_1$, $v_\phi = Uv_{\phi 1}$, $v_r \equiv U\psi v_{r1}$
 $v_\vartheta \equiv Uv_{\vartheta 1}$, $p = p_o p_1$, $p_o \equiv U\eta_o R/\varepsilon^2$ (7)

and Reynolds number, modified Reynolds number and Strouhal - Deborah number are as follows:

$$Re \equiv \rho U \epsilon / \eta_o$$
, $Re \psi \equiv \rho \omega \epsilon^2 / \eta_o$, $Str \equiv R/(Ut_o)$, $De \equiv \beta U/(\eta_o R)$ (8)

hence:

DeStr =
$$\beta/(\eta_0 t_0)$$
 = Des , Re ψ Str = $\rho \epsilon^2/(\eta_0 t_0)$ = Res (8a)

For the oil containing inhibitors the constant β/t_o is always $0 < \beta/t_o < \eta_o$ and of values usually in the range from 0.0001 to 0.1000 Pas². The dimensionless symbols have lower index "1". Hence the equations (1) \div (6) obtain the following dimensionless form :

$$\operatorname{Res} \frac{\partial v_{\phi 1}}{\partial t_{1}} = -\frac{1}{\sin \theta_{1}} \frac{\partial p_{1}}{\partial \phi} + \frac{\partial}{\partial r_{1}} \left(\frac{\partial v_{\phi 1}}{\partial r_{1}} \right) + \operatorname{Des} \frac{\partial^{3} v_{\phi 1}}{\partial t_{1} \partial r_{1}^{2}}$$
(9)

$$0 = \frac{\partial p_1}{\partial r_1} \tag{10}$$

$$\operatorname{Res} \frac{\partial v_{91}}{\partial t_{1}} = -\frac{\partial p_{1}}{\partial 9_{1}} + \frac{\partial}{\partial r_{1}} \left(\frac{\partial v_{91}}{\partial r_{1}} \right) + \operatorname{Des} \frac{\partial^{3} v_{91}}{\partial t_{1} \partial r_{1}^{2}}$$
(11)

$$\frac{\partial \mathbf{v}_{\phi 1}}{\partial \phi} + \sin(\vartheta_1) \frac{\partial \mathbf{v}_{r1}}{\partial r_1} + \frac{\partial}{\partial \vartheta_1} \left[\mathbf{v}_{\vartheta 1} \sin(\vartheta_1) \right] = 0 \tag{12}$$

where: $0 \le \varphi \le 2\pi c_1$, $0 \le c_1 \le 1$, $\pi/8 \le \vartheta_1 \le \pi/2$, $0 \le r_1 \le h_1$

THE METHOD OF INTEGRATION

For lubrication at impulsive motion a new dimensionless variable was introduced [4]:

$$\chi \equiv r_1 N , N \equiv \frac{1}{2} \sqrt{\frac{\text{Res}}{t_1}} , t_1 > 0 , 0 < \frac{\text{Des}}{t_1} < 1$$
 (13)

and the solutions in the form of the following convergent series, were assumed:

$$\mathbf{v}_{\phi 1} = \mathbf{v}_{\phi 0 \Sigma}(\chi, \phi, \vartheta_1) + \frac{\mathrm{Des}}{\mathsf{t}_1} \mathbf{v}_{\phi 1 \Sigma}(\chi, \phi, \vartheta_1) + \left(\frac{\mathrm{Des}}{\mathsf{t}_1}\right)^2 \mathbf{v}_{\phi 2 \Sigma}(\chi, \phi, \vartheta_1) + \dots$$
(14)

$$v_{\vartheta 1} = v_{\vartheta 0\Sigma}(\chi, \varphi, \vartheta_1) + \frac{Des}{t_1} v_{\vartheta 1\Sigma}(\chi, \varphi, \vartheta_1) + \left(\frac{Des}{t_1}\right)^2 v_{\vartheta 2\Sigma}(\chi, \varphi, \vartheta_1) + \dots$$
(15)

$$\mathbf{v}_{r1} = \mathbf{v}_{r0\Sigma}(\chi, \varphi, \vartheta_1) + \frac{\mathrm{Des}}{\mathsf{t}_1} \mathbf{v}_{r1\Sigma}(\chi, \varphi, \vartheta_1) + \left(\frac{\mathrm{Des}}{\mathsf{t}_1}\right)^2 \mathbf{v}_{r2\Sigma}(\chi, \varphi, \vartheta_1) + \dots$$
 (16)

$$p_{1} = p_{10}(\varphi, \vartheta_{1}, t_{1}) + \frac{Des}{t_{1}} p_{11}(\varphi, \vartheta_{1}, t_{1}) + \left(\frac{Des}{t_{1}}\right)^{2} p_{12}(\varphi, \vartheta_{1}, t_{1}) + \dots$$
(17)

where:
$$t_1 > 0$$
, $0 < Des << 1$, $(Des/t_1) < 1$

The first terms of the series (14), (17) describe oil flow parameters in impulsive unsteady motion, with neglecting the viscoelastic properties. The second, third, etc terms in the series (14), (17) describe the corrections of oil flow parameters, caused by the time-changeable viscoelastic oil properties. In (9) - (11) the derivatives with respect to the variables t_1 , r_1 , can be replaced by the derivatives with respect to the variable χ only, by using the following relationships:

$$\frac{\partial}{\partial t_{1}} = \frac{\partial}{\partial \chi} \frac{\partial \chi}{\partial t_{1}} = -\frac{1}{4} \sqrt{\operatorname{Res}} \frac{r_{1}}{t_{1} \sqrt{t_{1}}} \frac{\partial}{\partial \chi} = -\frac{\chi}{2t_{1}} \frac{\partial}{\partial \chi}$$

$$\frac{\partial^{2}}{\partial r_{1}^{2}} = \frac{\partial}{\partial r_{1}} \left(\frac{\partial}{\partial r_{1}}\right) = \frac{\partial}{\partial \chi} \left(\frac{\partial}{\partial \chi} \frac{\partial \chi}{\partial r_{1}}\right) \frac{\partial \chi}{\partial r_{1}} = \frac{\operatorname{Res}}{4t_{1}} \frac{\partial^{2}}{\partial \chi^{2}}$$
(18)

$$\frac{\partial^{3}}{\partial t_{1} \partial r_{1}^{2}} = \frac{\partial}{\partial t_{1}} \left(\frac{\operatorname{Res}}{4t_{1}} \frac{\partial^{2}}{\partial \chi^{2}} \right) = -\frac{\operatorname{Res}}{4t_{1}^{2}} \frac{\partial^{2}}{\partial \chi^{2}} + \frac{\operatorname{Res}}{4t_{1}} \frac{\partial}{\partial \chi} \left(\frac{\partial^{2}}{\partial \chi^{2}} \right) \frac{\partial \chi}{\partial t_{1}} = -\frac{\operatorname{Res}}{4t_{1}^{2}} \left(\frac{\partial^{2}}{\partial \chi^{2}} + \frac{\chi}{2} \frac{\partial^{3}}{\partial \chi^{3}} \right)$$
(19)

Next, the series (14)-(17) were put into the changed set of the equations (9)-(12) where the variables t_1, r_1 were replaced by the variable χ . And, the terms multiplied by the parameter $(Des/t_1)^k$ of the same power values k, for k = 0, 1, 2, ..., were respectively equalled to each other. Thus the following sequence of the sets of ordinary second-order differential equations, was obtained [5]:

$$(v_{i0\Sigma})^{(2)} + 2\chi(v_{i0\Sigma})^{(1)} = \frac{1}{N_i^2} \frac{\partial p_{10}}{\partial \alpha_i}$$
 (20)

$$(v_{i1\Sigma})^{(2)} + 2\chi(v_{i1\Sigma})^{(1)} + 4(v_{i1\Sigma}) = \frac{1}{N_i^2} \frac{\partial p_{11}}{\partial \alpha_i} + (v_{i0\Sigma})^{(2)} + \frac{1}{2}\chi(v_{i0\Sigma})^{(2)}$$
(21)

$$(v_{i2\Sigma})^{(2)} + 2\chi(v_{i2\Sigma})^{(1)} + 8(v_{i2\Sigma}) = \frac{1}{N_i^2} \frac{\partial p_{12}}{\partial \alpha_i} + 2(v_{i1\Sigma})^{(2)} + \frac{1}{2}\chi(v_{i1\Sigma})^{(3)}$$
(22)

and so on,

where :
$$i = \varphi$$
, ϑ , $\alpha_{\varphi} \equiv \varphi$, $\alpha_{\vartheta} \equiv \vartheta_1$

The upper indices: (1), (2), (3), ... denote: the first, second, third, etc. derivative with respect to the variable χ , and:

$$(N_{\Theta})^{2} \equiv N^{2} \sin(\vartheta_{1}) \quad , \quad N_{\vartheta} \equiv N$$
 (23)

GENERAL SOLUTIONS AND VALIDITY OF BOUNDARY CONDITIONS

The general solutions of the equations (20) for : $i = \varphi$, ϑ ; have the form :

$$v_{i0\Sigma}(\chi) = C_{i1}v_{01}(\chi) + C_{i2}v_{02}(\chi) + v_{i03}(\chi)$$
(24)

where : C_{i1} , C_{i2} - integration constants.

The following particular solutions of homogeneous and non - homogeneous differential equations were obtained:

$$v_{01}(\chi) = \int_{0}^{\chi} e^{-\chi_{1}^{2}} d\chi_{1}$$
, $v_{02}(\chi) = 1$ (25)

$$v_{i03}(\chi) = -\frac{1}{N_i^2} \frac{\partial p_{10}}{\partial \alpha_i} \left[\int_0^{\chi} e^{\chi_1^2} v_{01}(\chi_1) d\chi_1 - v_{01}(\chi) \int_0^{\chi} e^{\chi_1^2} d\chi_1 \right]$$
 (26)

where : $0 \le \gamma_1 \le \gamma \equiv r_1 N$.

For $t_1 \to 0$, $N \to \infty$, thus $\chi \to \infty$. For $t_1 \to \infty$, $N \to 0$ hence for $r_1 > 0$ will be $\chi \to 0$. For $t_1 > 0$ and $r_1 = 0$ also $\chi = 0$. The following limits are true:

$$\begin{split} v_{01}(\chi) &= \pi^{0.5}/2 \ \text{for} : \chi \to \infty \ , \ t_1 \to 0 \ , \ N \to \infty \\ v_{01}(\chi) &= 0 \quad \text{for} : \chi \to 0 \ , \ r_1 = 0 \ , \ 0 < t_1 < t_2 < \infty \ , \ N > 0 \\ v_{01}(\chi) &= 0 \quad \text{for} : \chi \to 0 \ , \ r_1 > 0 \ , \ t_1 \to \infty \ , \ N \to 0 \\ v_{i03}(\chi) &= 0 \quad \text{for} : \chi \to 0 \ , \ r_1 = 0 \ , \ 0 < t_1 < t_2 < \infty \ , \ N > 0 \ \text{where} : i = \phi \ , \vartheta \end{split} \tag{27}$$

$$v_{\phi 03}(\chi) &= -\frac{r_1^2}{2\sin \vartheta_1} \frac{\partial p_{10}}{\partial \phi} \quad \text{for} : \chi \to 0 \ , \ r_1 > 0 \ , \ t_1 \to \infty \ , \ N \to 0$$

$$v_{\vartheta 03}(\chi) &= -\frac{r_1^2}{2\sin \vartheta_1} \frac{\partial p_{10}}{\partial \varphi} \quad \text{for} : \chi \to 0 \ , \ r_1 > 0 \ , \ t_1 \to \infty \ , \ N \to 0 \end{split}$$

The spherical journal moves only in the circumferential direction φ . Hence the oil velocity components on the journal surface in this direction are equal to the peripheral velocity of the spherical journal surface. The oil velocity component on the spherical journal surface in the meridianal direction ϑ equals zero because the spherical journal is motionless in ϑ - direction. The oil flow around the journal is assumed viscous. Hence on the journal surface the oil velocity component in the gap height direction equals zero. Therefore the following boundary conditions are valid:

$$\begin{aligned} v_{\phi0\Sigma}(\chi=0) &= \sin\theta_1 \quad , \quad v_{\theta0\Sigma}(\chi=0) = 0 \quad , \quad v_{r0\Sigma}(\chi=0) = 0 \\ \text{for : } r_1 &= 0 \Leftrightarrow \chi = 0 \text{ and } 0 < t_1 < t_2 < \infty \quad , \quad N > 0 \end{aligned} \tag{28}$$

The spherical sleeve surface is motionless in both circumferenial and meridianal directions. But the spherical sleeve performs any impulsive displacements in the gap height direction. Hence the gap height changes along with time. Thus the oil velocity components on the sleeve surface are equal to zero in both circumferential and meridianal directions. The oil velocity component in the gap height direction r is equal to the first derivative of the gap height with respect to time. Hence the following boundary conditions are valid:

$$v_{\phi0\Sigma}(\chi=M) = 0 \quad , \quad v_{\theta0\Sigma}(\chi=M) = 0 \quad , \quad v_{r0\Sigma}(\chi=M) = Str\partial h_1/\partial t_1$$
 for : $r_1 \to h_1 \Leftrightarrow \chi \to Nh_1 \equiv M \quad , \quad 0 < t_1 < t_2 < \infty \quad , \quad N > 0$ (29)

where : $h = \varepsilon h_1$ - gap height , h_1 - dimensionless gap height , $Str = 1/\omega t_0$

Imposing the conditions (28), (29) on the solution (24) one obtains:

$$\begin{split} &C_{\phi 1} v_{01}(\chi=0) + C_{\phi 2} + v_{\phi 03}(\chi=0) = sin\vartheta_1 & \text{for : } r_1 = 0 \\ &C_{\phi 1} v_{01}(\chi=M) + C_{\phi 2} + v_{\phi 03}(\chi=M) = 0 & \text{for : } r_1 = h_1 \\ &C_{\vartheta 1} v_{01}(\chi=0) + C_{\vartheta 2} + v_{\vartheta 03}(\chi=0) = 0 & \text{for : } r_1 = 0 \\ &C_{\vartheta 1} v_{01}(\chi=M) + C_{\vartheta 2} + v_{\vartheta 03}(\chi=M) = 0 & \text{for : } r_1 = h_1 \end{split}$$

By taking into account the limits (27) the following solutions of the set of the equations (30), are obtained:

$$C_{\phi 1} = -\frac{\sin \theta_1 + v_{\phi 03}(M)}{v_{01}(M)} , C_{\theta 1} = -\frac{v_{\theta 03}(M)}{v_{01}} , C_{\phi 2} = \sin \theta_1 , C_{\theta 2} = 0$$
 (31)

Now, into the right hand side of (21) the solution (24), (25), (26), (31) is inserted.

Thus the general solution of (21) obtains the following form:

$$v_{i1\Sigma}(\chi) = C_{i3}v_{11}(\chi) + C_{i4}v_{12}(\chi) + v_{i13}(\chi)$$
 for : $i = \varphi$, ϑ (32)

where: C_{i3}, C_{i4} - integration constants.

The particular solutions are as follows:

$$v_{11}(\chi) = \chi e^{-\chi^2} , \quad v_{12}(\chi) = \chi e^{-\chi^2} \int_{\delta}^{\chi} \frac{1}{\chi_1^2} e^{-\chi_1^2} d\chi_1$$
 (33)

$$\begin{split} v_{i13}(\chi,C_{i1}) &= v_{11}(\chi) \int\limits_{0}^{\chi} \left\{ C_{i1} \, \chi_{1}(\chi_{1} + 2) - \left(1 + \frac{\chi_{1}}{2} \right) e^{\chi_{1}^{2}} \, \frac{d^{2}}{d\chi_{1}^{2}} \left[v_{i03}(\chi_{1}) \right] + \frac{1}{N_{i}^{2}} \frac{\partial p_{11}}{\partial \alpha_{i}} \right\} v_{12}(\chi_{1}) \, d\chi_{1} + \\ &+ v_{12}(\chi) \int\limits_{0}^{\chi} \left\{ \left(1 + \frac{\chi_{1}}{2} \right) e^{\chi_{1}^{2}} \, \frac{d^{2}}{d\chi_{1}^{2}} \left[v_{i03}(\chi_{1}) \right] + \frac{1}{N_{i}^{2}} \frac{\partial p_{11}}{\partial \alpha_{i}} - C_{i1}\chi_{1}(\chi_{1} + 2) \right\} v_{11}(\chi_{1}) \, d\chi_{1} \\ &\qquad \qquad \qquad \text{for : } i = \phi \,, \, \vartheta \quad , \quad 0 < \delta \leq \chi_{1} \leq \chi \end{split}$$

The solutions (32) represent the corrections of the oil velocity components due to the viscoelastic oil properties. By virtue of the solutions (33) and (34), for : $\chi \to 0$, $r_1 \to 0$, N > 0, it follows :

$$\lim_{\chi \to 0, N > 0} V_{12}(\chi) = \lim_{\chi \to 0, N > 0} \chi e^{-\chi^2} \int_{\delta}^{\chi} \frac{1}{\chi_1^2} e^{\chi_1^2} d\chi_1 = -1$$
(35)

The following limits are true:

$$\begin{array}{lllll} v_{11}(\chi) = 0 & & \text{for}: \chi \to 0 & , & r_1 = 0 & , & 0 < t_1 < t_2 < \infty & , & N > 0 \\ v_{12}(\chi) = -1 & & \text{for}: \chi \to 0 & , & r_1 = 0 & , & 0 < t_1 < t_2 < \infty & , & N > 0 \\ v_{i13}(\chi) = 0 & & \text{for}: \chi \to 0 & , & r_1 = 0 & , & 0 < t_1 < t_2 < \infty & , & N > 0 & \text{where}: i = \phi \,, \vartheta \end{array} \right. \eqno(36)$$

The corrections of oil velocity components can not violate the boundary conditions (28),(29) which are assumed on the journal and sleeve surfaces in the circumferential and meridianal directions. Therefore, the following boundary conditions were applied to the corrections of oil velocity components:

$$\begin{split} v_{\phi1\Sigma}(\chi=0) &= 0 \quad , \quad v_{\vartheta1\Sigma}(\chi=0) = 0 \quad \text{for} : r_1 = 0 \Leftrightarrow \chi = 0 \quad , \quad 0 < t_1 < t_2 < \infty \quad , \quad N > 0 \\ v_{\phi1\Sigma}(\chi=M) &= 0 \quad , \quad v_{\vartheta1\Sigma}(\chi=M) = 0 \quad \text{for} : r_1 \to h_1 \Leftrightarrow \chi \to Nh_1 \equiv M \quad , \quad 0 < t_1 < t_2 < \infty \quad , \quad N > 0 \end{split} \tag{37}$$
 Imposing conditions (37) on the general solution (32) one gets :

$$\begin{split} &C_{\phi 3} v_{11}(\chi=0) + C_{\phi 4} v_{21}(\chi=0) + v_{\phi 13}(\chi=0) = 0 \qquad \text{for : } r_1 = 0 \\ &C_{\phi 3} v_{11}(\chi=M) + C_{\phi 4} v_{21}(\chi=M) + v_{\phi 13}(\chi=M) = 0 \quad \text{for : } r_1 = h_1 \end{split} \tag{38}$$

$$C_{93}v_{11}(\chi = 0) + C_{94}v_{21}(\chi = 0) + v_{913}(\chi = 0) = 0 for : r_1 = 0$$

$$C_{93}v_{11}(\chi = M) + C_{94}v_{21}(\chi = M) + v_{913}(\chi = M) = 0 for : r_1 = h_1$$
(39)

By taking into account the limits (36), the following solutions of the set of the equations (38),(39) were obtained:

$$C_{i3} = \frac{v_{i13}(\chi = h_1 N)}{v_{11}(\chi = h_1 N)}, \quad C_{i4} = 0 \quad \text{for : } i = \phi, \vartheta$$
 (40)

 $\text{where}: 0 \leq \chi_1 \leq h_1 N \quad , \quad N = \frac{1}{2} \sqrt{\frac{Re \, \psi Str}{t_1}} \quad , \quad 0 < t_1 < \infty \quad , \quad 0 \leq r_1 \leq h_1 \quad , \quad b_{m1} \leq \vartheta_1 \leq b_{s1} \quad , \quad 0 < \phi < 2\pi c_1 \quad , \quad 0 \leq c_1 < \infty$

NEWTONIAN UNSTEADY LUBRICATION

By neglecting the viscoelastic properties of oil, by virtue of solutions (24) and constants (31), the particular velocity components of oil in φ - and ϑ - directions for non steady flow obtained the following dimensionless form :

$$\begin{split} v_{\phi0\Sigma}(\phi,r_{1},\vartheta_{1},t_{1}) &= \sin\vartheta_{1} - \left\{\sin\vartheta_{1} - \frac{\sqrt{\pi}}{2N^{2}\sin\vartheta_{1}} \frac{\partial p_{10}}{\partial \phi} \left[Y\left(\chi = Nh_{1}\right)\right]\right\} \cdot \frac{erf(r_{1}N)}{erf(h_{1}N)} + \\ &- \frac{\sqrt{\pi}}{2N^{2}\sin\vartheta_{1}} \frac{\partial p_{10}}{\partial \phi} Y\left(\chi = Nr_{1}\right) \end{split} \tag{41}$$

$$v_{\vartheta 0\Sigma}(\phi, r_{1}, \vartheta_{1}, t_{1}) = \frac{\sqrt{\pi}}{2N^{2}} \frac{\partial p_{10}}{\partial \vartheta_{1}} \left[Y\left(\chi = Nh_{1}\right) \right] \cdot \frac{erf\left(r_{1}N\right)}{erf\left(h_{1}N\right)} - \frac{\sqrt{\pi}}{2N^{2}} \frac{\partial p_{10}}{\partial \vartheta_{1}} Y\left(\chi = Nr_{1}\right)$$

$$(42)$$

where:

$$Y(\chi) = \int_{0}^{\chi} e^{\chi_{1}^{2}} \operatorname{erf} \chi_{1} d\chi_{1} - \operatorname{erf}(h_{1}N) \int_{0}^{\chi} e^{\chi_{1}^{2}} d\chi_{1}$$
(43)

$$N = \frac{1}{2} \sqrt{\frac{\text{Res}}{t_1}} , \text{ erf}(\chi_1) = \frac{2}{\sqrt{\pi}} \int_{0}^{\chi_1} e^{-\chi_2^2} d\chi_2$$
 (44)

for:

$$0 \leq t_1 < \infty \ , \ 0 \leq r_1 \leq h_1 \ , \ b_{m1} \leq \vartheta_1 \leq b_{s1} \ , \ 0 < \phi < 2\pi c_1 \ , \ 0 \leq c_1 < \infty \ , \ 0 \leq \chi_2 \leq \chi_1 \leq \chi \equiv r_1 N \leq h_1 N \equiv M \ , \ h_1 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_2 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_3 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_4 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_5 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_7 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_8 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_8 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_8 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_8 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_8 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_8 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_9 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_9 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_9 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_9 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_9 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_9 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_9 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_9 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_9 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_9 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_9 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_9 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_9 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_9 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_9 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_9 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_9 = h_1(\phi, \vartheta_1, t_1) \leq h_1 N \equiv M \ , \ h_9 = h_1(\phi, \vartheta_1, t_1) \leq h_1(\phi, \vartheta_1, t_1)$$

The oil velocity components (41),(42) were put into the continuity equation (12) and both sides of this equation were integrated with respect to the variable r_1 . The oil velocity component $v_{r0\Sigma}$ in the gap height direction equals zero on the spherical journal surface. Therefore by imposing the boundary condition $v_{r0\Sigma} = 0$ for $r_1 = 0$, the oil velocity component in the gap height direction obtained the following form :

$$\begin{split} v_{r0\Sigma}(\phi,r_{1},\vartheta_{1},t_{1}) &= -\frac{Ne^{-h_{1}^{2}N^{2}}}{erf\left(h_{1}N\right)} \left[\frac{\partial h_{1}}{\partial \phi} - \frac{\sqrt{\pi}}{2} \left(\frac{1}{\sin^{2}\vartheta_{1}} \frac{\partial h_{1}}{\partial \phi} \frac{\partial p_{10}}{\partial \phi} + \frac{\partial h_{1}}{\partial \vartheta_{1}} \frac{\partial p_{10}}{\partial \vartheta_{1}} \right) \frac{1}{N} \int_{0}^{h_{1}N} e^{\chi^{2}} erf\chi d\chi \right]_{0}^{r_{1}} \frac{erf\left(r_{2}N\right)}{erf\left(h_{1}N\right)} dr_{2} + \\ &- \frac{\sqrt{\pi}}{2} \left(\frac{1}{\sin^{2}\vartheta_{1}} \frac{\partial^{2}p_{10}}{\partial \phi^{2}} + \frac{\partial^{2}p_{10}}{\partial \vartheta_{1}^{2}} + \frac{\partial p_{10}}{\partial \vartheta_{1}} \cot \vartheta_{1} \right) \left\{ \frac{1}{N^{2}} Y\left(\chi = h_{1}N\right) \int_{0}^{r_{1}} \frac{erf\left(r_{2}N\right)}{erf\left(h_{1}N\right)} dr_{2} - \int_{0}^{r_{1}} Y\left(\chi = r_{2}N\right) dr_{2} \right\} \end{split} \tag{45}$$

$$\text{where}: 0 \leq t_1 < \infty \quad , \quad 0 \leq r_2 \leq r_1 \leq h_1 \quad , \quad b_{m1} \leq \vartheta_1 \leq b_{s1} \quad , \quad 0 < \phi < 2\pi c_1 \quad , \quad 0 \leq c_1 < \infty \quad , \quad 0 \leq \chi_2 \leq \chi_1 \leq \chi \equiv r_1 N \leq h_1 N \equiv M$$

The oil velocity component $v_{r0\Sigma}$ in the gap height direction does not equal zero on the sleeve surface. Therefore by integrating the continuity equation (12) and imposing the boundary condition (29) for r_1 = h_1 on the velocity component in the gap height direction, the following equation was obtained:

$$\frac{\partial}{\partial \varphi} \int_{0}^{h_{1}} v_{\varphi 0 \Sigma} dr_{1} + \frac{\partial}{\partial \vartheta_{1}} \int_{0}^{h_{1}} \sin \vartheta_{1} v_{\vartheta 0 \Sigma} dr_{1} = -\operatorname{Str} \frac{\partial h_{1}}{\partial t_{1}} \sin \vartheta_{1}$$

$$\tag{46}$$

If the expressions (41)÷(42) are put into (46) the following modified Reynolds equation is yielded:

$$\frac{\sqrt{\pi}}{2N^2} \frac{1}{\sin \theta_1} \frac{\partial}{\partial \phi} \left\{ \begin{bmatrix} \int_0^{h_1} \operatorname{erf}(r_1 N) dr_1 \\ \frac{1}{2} \operatorname{erf}(h_1 N) \end{bmatrix} Y(\chi = Nh_1) - \int_0^{h_1} Y(\chi = Nr_1) dr_1 \right\} + \frac{1}{2} \frac{\partial}{\partial \phi} \left\{ \int_0^{h_1} \operatorname{erf}(h_1 N) dr_1 \\ \frac{\partial}{\partial \phi} \operatorname{erf}(h_1 N) \right\} + \frac{1}{2} \frac{\partial}{\partial \phi} \left\{ \int_0^{h_1} \operatorname{erf}(r_1 N) dr_1 \\ \frac{\partial}{\partial \phi} \operatorname{erf}(h_1 N) \right\} + \frac{1}{2} \frac{\partial}{\partial \phi} \left\{ \int_0^{h_1} \operatorname{erf}(r_1 N) dr_1 \\ \frac{\partial}{\partial \phi} \operatorname{erf}(h_1 N) \right\} + \frac{1}{2} \frac{\partial}{\partial \phi} \left\{ \int_0^{h_1} \operatorname{erf}(r_1 N) dr_1 \\ \frac{\partial}{\partial \phi} \operatorname{erf}(h_1 N) \\ \frac{\partial}{\partial \phi} \operatorname{erf}(h_1 N) \right\} + \frac{1}{2} \frac{\partial}{\partial \phi} \left\{ \int_0^{h_1} \operatorname{erf}(r_1 N) dr_1 \\ \frac{\partial}{\partial \phi} \operatorname{erf}(h_1 N) \\ \frac{\partial}{\partial \phi} \operatorname{erf}(h$$

$$+\frac{\sqrt{\pi}}{2N^{2}}\frac{\partial}{\partial\theta_{1}}\left\{\left[\frac{\int_{0}^{h_{1}}\operatorname{erf}(r_{1}N)dr_{1}}{\operatorname{erf}(h_{1}N)}Y(\chi=Nh_{1})-\int_{0}^{h_{1}}Y(\chi=Nr_{1})dr_{1}\right]\frac{\partial p_{10}}{\partial\theta_{1}}\sin\theta_{1}\right\}=\tag{47}$$

$$= -\left(\sin \theta_{1}\right) \frac{\partial}{\partial \phi} \left(\int_{0}^{h_{1}} \left[1 - \frac{\operatorname{erf}(r_{1}N)}{\operatorname{erf}(h_{1}N)} \right] dr_{1} \right) - \operatorname{Str} \frac{\partial h_{1}}{\partial t_{1}} \left(\sin \theta_{1}\right)$$

$$\begin{array}{c} \text{where :} \\ 0 \leq r_2 \leq r_1 \leq h_1 \quad , \quad 0 \leq \phi < 2\pi c_1 \quad , \quad 0 \leq c_1 < 1 \quad , \quad 0 \leq \vartheta_1 < \pi/2 \quad , \quad 0 \leq t_1 < \infty \\ 0 \leq \chi_2 \leq \chi_1 \leq h_1 N \quad , \quad 0 \leq N(t_1) = 0.5 (Res/t_1)^{0.5} < \infty \end{array}$$

The modified Reynolds equation (47) determines an unknown pressure function $p_{10}(\varphi, \vartheta_1, t_1)$. If t_1 tends to infinity, i.e. N tends to zero, then the equation (47) tends to the classical Reynolds equation. To explain this fact the following limits were calculated:

$$\lim_{N\to 0}\frac{\sqrt{\pi}}{2N^2}\,Y\Big(\chi=h_1N\Big)\equiv\lim_{N\to 0}\frac{\sqrt{\pi}}{2N^2}\Bigg[\int\limits_0^{h_1N}exp(\chi^2)erf(\chi)d\chi-erf(h_1N)\int\limits_0^{h_1N}exp(\chi^2)d\chi\Bigg]=$$

$$= \lim_{N \to 0} \frac{1}{N^2} \left\{ \int_0^{h_1 N} \left[\exp(\chi^2) \int_0^{\chi} \exp(-\chi_1^2) d\chi_1 \right] d\chi - \left(\int_0^{h_1 N} \exp(-\chi^2) d\chi \right) \left(\int_0^{h_1 N} \exp(\chi^2) d\chi \right) \right\} =$$
(48)

$$= -\lim_{N\to 0} \frac{h_1 \int_0^{Nh_1} \exp(\chi^2) d\chi}{2N \exp(h_1^2 N^2)} = -\frac{h_1^2}{2} \lim_{N\to 0} \frac{\exp(h_1^2 N^2)}{\exp(h_1^2 N^2) + 2h_1^2 N^2 \exp(h_1^2 N^2)} = -\frac{h_1^2}{2}$$

and, analogously:

$$\lim_{N \to 0} \frac{\sqrt{\pi}}{2N^2} Y(\chi = Nr_1) = -\frac{r_1^2}{2}$$
(49)

$$\lim_{N \to 0} \frac{\operatorname{erf}(r_1 N)}{\operatorname{erf}(h_1 N)} = \frac{r_1}{h_1}$$
(50)

Thus the equation (46) for $N \rightarrow 0$ tends to the following form :

$$\begin{split} &\frac{1}{\sin\vartheta_{1}}\frac{\partial}{\partial\varphi}\left\{\!\!\left[\!\left(-\frac{h_{1}^{2}}{2}\right)\!\!\int_{0}^{h_{1}}\frac{r_{1}}{h_{1}}\,dr_{1}-\int_{0}^{h_{1}}\!\left(-\frac{r_{1}^{2}}{2}\right)\!dr_{1}\right]\!\frac{\partial\,p_{10}}{\partial\varphi}\right\}+\frac{\partial}{\partial\vartheta_{1}}\!\left\{\!\!\left[\!\left(-\frac{h_{1}^{2}}{2}\right)\!\sin\vartheta_{1}\int_{0}^{h_{1}}\frac{r_{1}}{h_{1}}dr_{1}+\right.\right.\\ &\left.-\int_{0}^{h_{1}}\!\left(-\frac{r_{1}^{2}}{2}\right)\!\sin\vartheta_{1}dr_{1}\right]\!\frac{\partial\,p_{10}}{\partial\vartheta_{1}}\right\}=-(\sin\vartheta_{1})\frac{\partial}{\partial\varphi}\!\left[\int_{0}^{h_{1}}\!\left(1-\frac{r_{1}}{h_{1}}\right)\!dr_{1}\right]\!-Str\frac{\partial\,h_{1}}{\partial t_{1}}(\sin\vartheta_{1}) \end{split} \tag{51}$$

Finally, after calculations, the following form of the classical Reynolds equations of flow in the spherical coordinates was obtained:

$$\frac{1}{\sin \theta_1} \frac{\partial}{\partial \varphi} \left(h_1^3 \frac{\partial p_{10}}{\partial \varphi} \right) + \frac{\partial}{\partial \theta_1} \left(h_1^3 \frac{\partial p_{10}}{\partial \theta_1} \sin \theta_1 \right) = 6 \frac{\partial h_1}{\partial \varphi} \sin \theta_1 + 12 \operatorname{Str} \frac{\partial h_1}{\partial t_1} \sin \theta_1$$
 (52)

for:
$$0 \le \varphi < 2\pi c_1$$
, $0 \le c_1 < 1$, $0 \le \vartheta_1 < \pi/2$

The time-dependent average gap height with perturbations has the following form:

$$h_1 = (h_0/\epsilon)[1 + s_1 \cdot \exp(-t_0 t_1 \omega_0)]$$
where:
(53)

$$\begin{split} h_{o}(\phi,\vartheta_{1}) &\equiv \Delta\epsilon_{1}cos\phi \ sin\vartheta_{1} + \Delta\epsilon_{2} \ sin\phi \ sin\vartheta_{1} - \Delta\epsilon_{3} \ cos\vartheta_{1} - R \ + \\ &+ \left[\left(\Delta\epsilon_{1}cos\phi \ sin\vartheta_{1} + \Delta\epsilon_{2} \ sin\phi \ sin\vartheta_{1} - \Delta\epsilon_{3} \ cos\vartheta_{1} \right)^{2} + \left(R + \epsilon_{min}\right) \left(R + 2D + \epsilon_{min}\right) \right]^{0.5} \end{split} \tag{54}$$

The coefficient s controls changes of the gap height during the impulsive motion. If s > 0 the gap height increases, if s < 0, the gap height decreases. The symbol ω_0 denotes an angular velocity expressed in $[s^{-1}]$, describing impulsive changes of perturbations in the unsteady oil flow in the bearing gap in its height direction. If Strouhal number tends to zero the equation (52) tends to the classical Reynolds equation for stationary flow.

The centre of the spherical journal was assumed in the point O(0,0,0) and the centre of the spherical sleeve in the point $O_s(x-\Delta\epsilon_1,y-\Delta\epsilon_2,z+\Delta\epsilon_3)$. The eccentricity was determined by the value D (Fig.3). The lubrication region Ω indicated in Fig.3 was defined as follows: $0 \le \varphi \le \pi$, $\pi R/8 \le \alpha_3 \equiv \vartheta \le \pi R/2$.

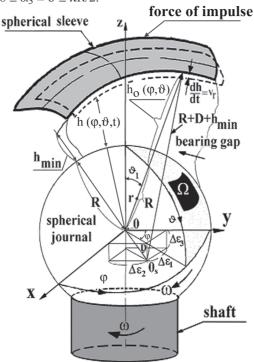


Fig.3. Schematic diagram of the gap height and eccentricities

VISCOELASTIC UNSTEADY EFFECTS

The particular corrections (32) of the oil velocity components in ϕ – and ϑ_1 – directions, caused by the viscoelastic, fluid properties and unsteady fluid flow, were multiplied by the factor DeStr/t₁. By using the expressions (32), (33), (34), (40) and boundary conditions (37), the corrections of the oil velocity components (32) obtained the following form :

$$\begin{split} \frac{\mathrm{Des}}{t_{1}} v_{\varphi l \Sigma}(\varphi, \theta_{1}, r_{1}, t_{1}) &= \frac{4\beta}{\rho \epsilon^{2}} \frac{N r_{l} e^{-r_{l}^{2} N^{2}}}{\sin \theta_{1}} \left\{ \frac{\partial p_{11}}{\partial \varphi} \begin{bmatrix} h_{l}^{h} \chi Y_{1}(\chi) \, d\chi + \\ \frac{N^{2}}{2} \left(h_{1}^{2} - r_{1}^{2} \right) Y_{1}(\chi = h_{1} N) \end{bmatrix} + \frac{\partial p_{10}}{\partial \varphi} \begin{bmatrix} h_{l}^{h} \chi Y_{2}(\chi) \, d\chi - Y_{1}(\chi = h_{1} N) \int_{0}^{h_{l} N} \chi e^{-\chi^{2}} Y_{2}(\chi) \, d\chi + \\ + Y_{1}(\chi = r_{1} N) \int_{0}^{r_{l} N} \chi e^{-\chi^{2}} Y_{2}(\chi) \, d\chi \end{bmatrix} - \frac{2}{\sqrt{\pi} \mathrm{erf}(h_{1} N)} \frac{\partial p_{10}}{\partial \varphi} \begin{bmatrix} h_{l}^{h} \chi Y_{1}(\chi) Y_{3}(\chi) Y(\chi) \, d\chi + \\ - Y_{1}(\chi = h_{1} N) \int_{0}^{h_{1} N} Y_{3}(\chi) Y(\chi) \, d\chi + Y_{1}(\chi = r_{1} N) \int_{0}^{r_{1} N} Y_{3}(\chi) Y(\chi) \, d\chi \end{bmatrix} \right\} + \\ - \frac{8\beta N^{2} r_{l} e^{-r_{l}^{2} N^{2}} \sin \theta_{1}}{\sqrt{\pi} \rho \epsilon^{2} \mathrm{erf}(h_{1} N)} \begin{bmatrix} Y_{1}(\chi = h_{1} N) \int_{0}^{h_{1} N} Y_{3}(\chi) \, d\chi - Y_{1}(\chi = r_{1} N) \int_{0}^{r_{1} N} Y_{3}(\chi) \, d\chi - \int_{r_{1} N}^{h_{1} N} Y_{1}(\chi) \, d\chi + \frac{N^{2}}{2} \left(h_{1}^{2} - r_{1}^{2} \right) Y_{1} \left[(\chi = h_{1} N) \right] \right] + \\ + \frac{\partial p_{10}}{\partial \theta_{1}} \begin{bmatrix} h_{1}^{h} \chi_{2}(\chi) \, d\chi - Y_{1}(\chi = h_{1} N) \int_{0}^{h_{1} N} \chi e^{-r^{2}} Y_{2}(\chi) \, d\chi + Y_{1}(\chi = r_{1} N) \int_{0}^{r_{1} N} \chi e^{-r^{2}} Y_{2}(\chi) \, d\chi \right] + \\ \frac{2}{\sqrt{\pi} \mathrm{erf}(h_{1} N)} \frac{\partial p_{10}}{\partial \theta_{1}} \begin{bmatrix} h_{1}^{h} \chi_{1}(\chi) Y_{3}(\chi) Y(\chi) \, d\chi - Y_{1}(\chi = h_{1} N) \int_{0}^{h_{1} N} \chi e^{-r^{2}} Y_{2}(\chi) \, d\chi + Y_{1}(\chi = r_{1} N) \int_{0}^{r_{1} N} \chi e^{-r^{2}} Y_{2}(\chi) \, d\chi \right] \right\}$$

$$Y_{1}(\chi) = \int_{\delta}^{\chi} \frac{1}{\chi_{1}^{2}} e^{\chi_{1}^{2}} d\chi_{1} , \quad Y_{2}(\chi) = \left(\chi + \frac{\chi^{2}}{2}\right) \left(2\chi e^{-\chi^{2}} \int_{0}^{\chi} e^{\chi_{1}^{2}} d\chi_{1} - 1\right) , \quad Y_{3}(\chi) = \chi^{2}(\chi + 2)e^{-\chi^{2}}$$
 (57)

 $\text{whereas}: 0 \leq t_1 < \infty \quad , \quad 0 \leq r_2 \leq r_1 \leq h_1 \quad , \quad b_{m1} \leq \vartheta_1 \leq b_{s1} \quad , \quad 0 < \phi < 2\pi c_1 \quad , \quad 0 \leq c_1 < \infty \quad , \quad 0 \leq \chi_1 \leq \chi \equiv r_1 N \leq h_1 N \equiv M_1 = M_2 + M_2 = M_1 + M_2 = M_2 + M_2 = M_1 + M_2 = M_2 + M_2 = M_2 = M_1 + M_2 = M$

The corrections of the oil velocity components (55), (56) were put into the continuity equation (12) and both sides of this equation were integrated with respect to the variable r₁. From the viscous oil properties it follows that the corrections of the oil velocity components in the gap height direction equal zero on the journal surface for $r_1 = 0$. Thus the corrections of oil velocity components in the gap height direction obtained the form:

$$v_{r1\Sigma}(\phi, \theta_1, r_1, t_1) = -\frac{1}{\sin \theta_1} \frac{\partial}{\partial \phi} \left(\int_{0}^{r_1} v_{\phi1\Sigma}(\phi, \theta_1, r_1, t_1) dr_1 \right) - \frac{1}{\sin \theta_1} \frac{\partial}{\partial \theta_1} \left(\int_{0}^{r_1} (\sin \theta_1) v_{\theta1\Sigma}(\phi, \theta_1, r_1, t_1) dr_1 \right)$$
(58)

The corrections of the oil velocity components can not violate the boundary conditions (28), (29)

assumed on the journal and sleeve surfaces in the gap height direction.

Hence the corrections of the oil velocity component in the gap height direction equal zero on the sleeve surface for $r_1 = h_1$.

By imposing this condition on the solution (58) the following modified Reynolds equation was obtained:

$$\begin{split} \frac{1}{\sin \vartheta_{1}} \frac{\partial}{\partial \varphi} \left\{ \frac{\partial p_{11}}{\partial \varphi} \begin{bmatrix} \int_{0}^{h_{1}} r_{1} e^{-r_{1}^{2}N^{2}} Z_{1}(r_{1}) dr_{1} + Z_{2}(h_{1}) \end{bmatrix} \right\} + \frac{\partial}{\partial \vartheta_{1}} \left\{ \frac{\partial p_{11}}{\partial \vartheta_{1}} \sin \vartheta_{1} \begin{bmatrix} \int_{0}^{h_{1}} r_{1} e^{-r_{1}^{2}N^{2}} Z_{1}(r_{1}) dr_{1} + Z_{2}(h_{1}) \end{bmatrix} \right\} = \\ &= \frac{2N \sin \vartheta_{1}}{erf(Nh_{1})} \int_{0}^{h_{1}} r_{1} e^{-r_{1}^{2}N^{2}} Z_{3}(r_{1}) dr_{1} + \frac{2}{\sqrt{\pi}erf(h_{1}N)\sin \vartheta_{1}} \frac{\partial}{\partial \varphi} \left\{ \frac{\partial p_{10}}{\partial \varphi} \begin{bmatrix} \int_{0}^{h_{1}} r_{1} e^{-r_{1}^{2}N^{2}} Z_{4}(r_{1}) dr_{1} - Z_{5}(h_{1}) \end{bmatrix} \right\} + \\ &+ \frac{2}{\sqrt{\pi}erf(h_{1}N)} \frac{\partial}{\partial \vartheta_{1}} \left\{ \frac{\partial p_{10}}{\partial \vartheta_{1}} \sin \vartheta_{1} \begin{bmatrix} \int_{0}^{h_{1}} r_{1} e^{-r_{1}^{2}N^{2}} Z_{4}(r_{1}) dr_{1} - Z_{5}(h_{1}) \end{bmatrix} \right\} + \\ &- \frac{1}{\sin \vartheta_{1}} \frac{\partial}{\partial \varphi} \left\{ \frac{\partial p_{10}}{\partial \varphi} \begin{bmatrix} \int_{0}^{h_{1}} r_{1} e^{-r_{1}^{2}N^{2}} Z_{6}(r_{1}) dr_{1} + Z_{7}(h_{1}) \end{bmatrix} \right\} - \frac{\partial}{\partial \vartheta_{1}} \left\{ \frac{\partial p_{10}}{\partial \vartheta_{1}} \sin \vartheta_{1} \begin{bmatrix} \int_{0}^{h_{1}} r_{1} e^{-r_{1}^{2}N^{2}} Z_{6}(r_{1}) dr_{1} + Z_{7}(h_{1}) \end{bmatrix} \right\} - \frac{\partial}{\partial \vartheta_{1}} \left\{ \frac{\partial p_{10}}{\partial \vartheta_{1}} \sin \vartheta_{1} \begin{bmatrix} \int_{0}^{h_{1}} r_{1} e^{-r_{1}^{2}N^{2}} Z_{6}(r_{1}) dr_{1} + Z_{7}(h_{1}) \end{bmatrix} \right\} - \frac{\partial}{\partial \vartheta_{1}} \left\{ \frac{\partial p_{10}}{\partial \vartheta_{1}} \sin \vartheta_{1} \begin{bmatrix} \int_{0}^{h_{1}} r_{1} e^{-r_{1}^{2}N^{2}} Z_{6}(r_{1}) dr_{1} + Z_{7}(h_{1}) \end{bmatrix} \right\} - \frac{\partial}{\partial \vartheta_{1}} \left\{ \frac{\partial p_{10}}{\partial \vartheta_{1}} \sin \vartheta_{1} \begin{bmatrix} \int_{0}^{h_{1}} r_{1} e^{-r_{1}^{2}N^{2}} Z_{6}(r_{1}) dr_{1} + Z_{7}(h_{1}) \end{bmatrix} \right\} - \frac{\partial}{\partial \vartheta_{1}} \left\{ \frac{\partial p_{10}}{\partial \vartheta_{1}} \sin \vartheta_{1} \begin{bmatrix} \int_{0}^{h_{1}} r_{1} e^{-r_{1}^{2}N^{2}} Z_{6}(r_{1}) dr_{1} + Z_{7}(h_{1}) \end{bmatrix} \right\} - \frac{\partial}{\partial \vartheta_{1}} \left\{ \frac{\partial p_{10}}{\partial \vartheta_{1}} \sin \vartheta_{1} \begin{bmatrix} \int_{0}^{h_{1}} r_{1} e^{-r_{1}^{2}N^{2}} Z_{6}(r_{1}) dr_{1} + Z_{7}(h_{1}) \end{bmatrix} \right\} - \frac{\partial}{\partial \vartheta_{1}} \left\{ \frac{\partial p_{10}}{\partial \vartheta_{1}} \sin \vartheta_{1} \begin{bmatrix} \int_{0}^{h_{1}} r_{1} e^{-r_{1}^{2}N^{2}} Z_{6}(r_{1}) dr_{1} + Z_{7}(h_{1}) \end{bmatrix} \right\} - \frac{\partial}{\partial \vartheta_{1}} \left\{ \frac{\partial p_{10}}{\partial \vartheta_{1}} \sin \vartheta_{1} \begin{bmatrix} \int_{0}^{h_{1}} r_{1} e^{-r_{1}^{2}N^{2}} Z_{6}(r_{1}) dr_{1} + Z_{7}(h_{1}) \right\} \right\} - \frac{\partial}{\partial \vartheta_{1}} \left\{ \frac{\partial p_{10}}{\partial \vartheta_{1}} \left\{ \frac{\partial p_{10}}{\partial \vartheta_{1}} \right\} \right\} + \frac{\partial}{\partial \vartheta_{1}} \left\{ \frac{\partial p_{10}}{\partial \vartheta_{1}} \left\{ \frac{\partial p_{10}}{\partial \vartheta_{1}} \right\} \right\} + \frac{\partial}{\partial \vartheta_{1}} \left\{ \frac{\partial p_{10}}{\partial \vartheta_{1}} \left\{ \frac{\partial p_{10}}{\partial \vartheta_{1}} \right\} \right\} \right\} + \frac{\partial$$

$$Z_{1}(\mathbf{r}_{1}) \equiv \int_{\mathbf{r}_{1}N}^{\mathbf{h}_{1}N} Y_{1}(\chi) d\chi - \mathbf{r}_{1}^{2} Y_{1}(\chi = \mathbf{h}_{1}N) , \quad Z_{2}(\mathbf{h}_{1}) = \frac{1}{4} (1 - e^{-\mathbf{h}_{1}^{2}N^{2}}) \mathbf{h}_{1}^{2} Y_{1}(\chi = N\mathbf{h}_{1})$$
 (60)

$$Z_{3}(r_{1}) \equiv Y_{1}(\chi = r_{1}N) \int_{0}^{h_{1}N} Y_{3}(\chi) d\chi - Y_{1}(\chi = r_{1}N) \int_{0}^{r_{1}N} Y_{3}(\chi) d\chi - \int_{r_{1}N}^{h_{1}N} Y_{1}(\chi) Y_{3}(\chi) d\chi$$
 (61)

$$Z_{4}(\mathbf{r}_{1}) = \int_{\mathbf{r}_{1}N}^{\mathbf{h}_{1}N} Y_{1}(\chi) Y_{3}(\chi) Y(\chi) d\chi + Y_{1}(\chi = \mathbf{r}_{1}N) \int_{0}^{\mathbf{r}_{1}N} Y_{1}(\chi) Y_{3}(\chi) Y(\chi) d\chi$$

$$Z_{5}(\mathbf{h}_{1}) \equiv \frac{1 - e^{-\mathbf{h}_{1}^{2}N^{2}}}{2N^{2}} Y_{1}(\chi = \mathbf{h}_{1}N) \int_{0}^{\mathbf{h}_{1}N} Y_{1}(\chi) Y_{3}(\chi) Y(\chi) d\chi$$
(62)

$$Z_{5}(h_{1}) = \frac{1 - e^{-h_{1}N}}{2N^{2}} Y_{1}(\chi = h_{1}N) \int_{0}^{h_{1}N} Y_{1}(\chi) Y_{3}(\chi) Y(\chi) d\chi$$
 (63)

$$Z_{6}(r_{1}) = \int_{r_{1}N}^{h_{1}N} Y_{2}(\chi) d\chi + Y_{1}(\chi = r_{1}N) \int_{0}^{h_{1}N} Y_{2}(\chi) \chi e^{-\chi^{2}} d\chi$$
 (64)

$$Z_7(h_1) = \frac{1 - e^{-h_1^2 N^2}}{2N^2} Y_1(\chi = h_1 N) \int_0^{h_1 N} Y_2(\chi) \chi e^{-\chi^2} d\chi$$
 (65)

The modified Reynolds equation (59) determines an unknown function $p_{11}(\varphi, \vartheta_1, t_1)$ of the pressure corrections due to the viscoelastic properties of oil in unsteady conditions.

NUMERICAL CALCULATIONS

The dimensionless pressure distribution p_{10} and its dimensionless corrections p_{11} , p_{12} , ... are determined in the lubrication region Ω by virtue of the modified Reynolds equations (47), (59) and by taking into account the gap height (53), (54). On the boundary of the region Ω the dimensional pressure and its corrections have values of the atmospheric pressure p_{at} . The region Ω indicated as a section of the bowl of the sphere (Fig. 3), is defined by the following inequalities: $\Omega: 0 \le \varphi \le \pi, \pi R/8 \le \vartheta \le \pi R/2$.

Numerical calculations were performed for:

- \bullet the radius of spherical journal R = 0.08 [m]
- the angular velocity of the perturbations of bearing sleeve $\omega_0 = 0.2 \text{ [s}^{-1}\text{]}$

- \bullet the characteristic time $t_0 = 0.001$ [s]
- \bullet the characteristic value of the radial clearance $\Psi \equiv \varepsilon/R = 0.001$.

And, the following bearing eccentricities $\Delta\epsilon_1{=}20~[\mu m]$, $\Delta\epsilon_2{=}~2~[\mu m]$, $\Delta\epsilon_3{=}~1[\mu m]$, the oil viscosity $\eta_o=0.03~[Pas]$, the pseudoviscosity coefficient $\beta=0.0006~[Pas^2]$, the oil density $\rho=950~[kg/m^3]$, the rotational velocity of the spherical journal, n=1500~[rev/min], and the average minimum gap height $\epsilon_{min}=4~[\mu m]$, were assumed. The numerical calculations were performed by using the Mathcad 11 Program and the finite difference method. The obtained

The numerical calculations were performed by using the Mathcad 11 Program and the finite difference method. The obtained pressure distributions for the dimensionless time values $t_1 = 1$, $t_1 = 10$, $t_1 = 100$, $t_1 = 1000$, $t_1 = 10000$, $t_1 = \infty$, and $t_1 = 10000$, and $t_2 = 10000$, $t_3 = 10000$, and $t_4 = 10000$, $t_5 = 10000$, and $t_5 = 10000$, and $t_5 = 10000$, $t_6 = 10000$, $t_7 = 10000$, $t_7 = 10000$, and $t_7 = 10000$, $t_7 = 10000$, and $t_7 = 10000$, $t_7 = 10000$, and $t_7 = 10000$, $t_7 = 10000$, $t_7 = 10000$, and $t_7 = 10000$, $t_7 = 10000$, $t_7 = 10000$, $t_7 = 10000$, and $t_7 = 10000$, $t_7 = 10000$, $t_7 = 10000$, and $t_7 = 10000$, and $t_7 = 10000$, $t_7 = 10000$, $t_7 = 10000$, $t_7 = 10000$, and $t_7 = 10000$, $t_7 = 10000$,

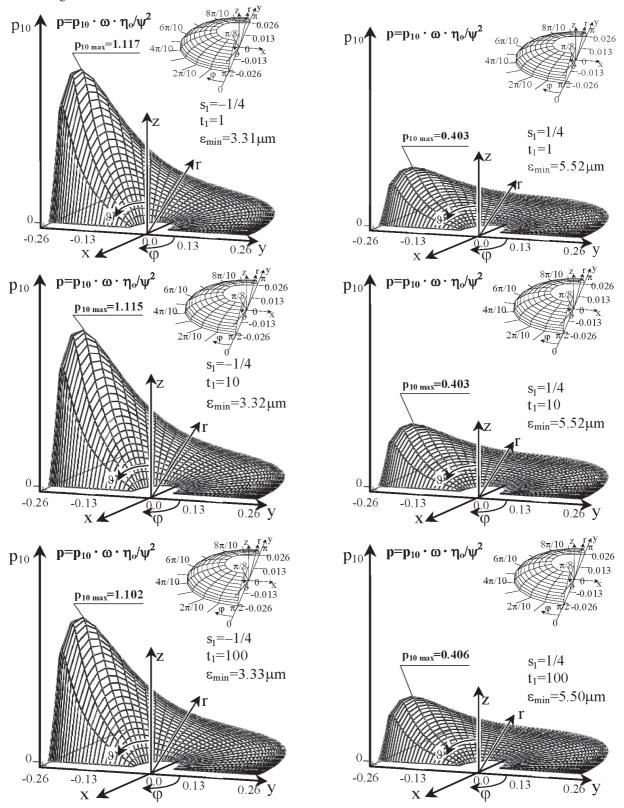


Fig.4. The dimensionless hydrodynamic pressure distributions inside the gap of slide spherical bearing, over the region $\Omega: 0 \le \varphi \le \pi$, $\pi R/8 \le \vartheta \le \pi R/2$, at the dimensionless time values : $t_1 = 1$, $t_1 = 10$, $t_1 = 100$, up to the impulse occurrence, for the increasing (decreasing) effects of the gap height changes, shown in the right (left) hand side column of the diagrams, respectively

In order to obtain real values of time it is necessary to multiply the dimensionless values t_1 by the characteristic time $t_0 = 0.001$ s. For example $t_1 = 1000$ denotes the time of 1s after impulse occurrence. In order to obtain realistic dimensional pressure values the dimensionless pressure values indicated in Fig.4 and 5 are to be multiplied by the dimensional coefficient $UR\eta_0/\epsilon^2$.

The pressure distributions shown on the right-hand sides of Fig.4 and 5 were obtained for the increasing of the gap height, caused by the impulse effects. In this case the longer the time up to the impulse, the more gap height decreases and pressure increases. The pressure distributions shown on the left-hand side of Fig.4 and 5 were obtained for the decreasing of the gap height, caused by the impulse effects. In this case the longer the time up to the impulse, the more gap height increases and pressure decreases.

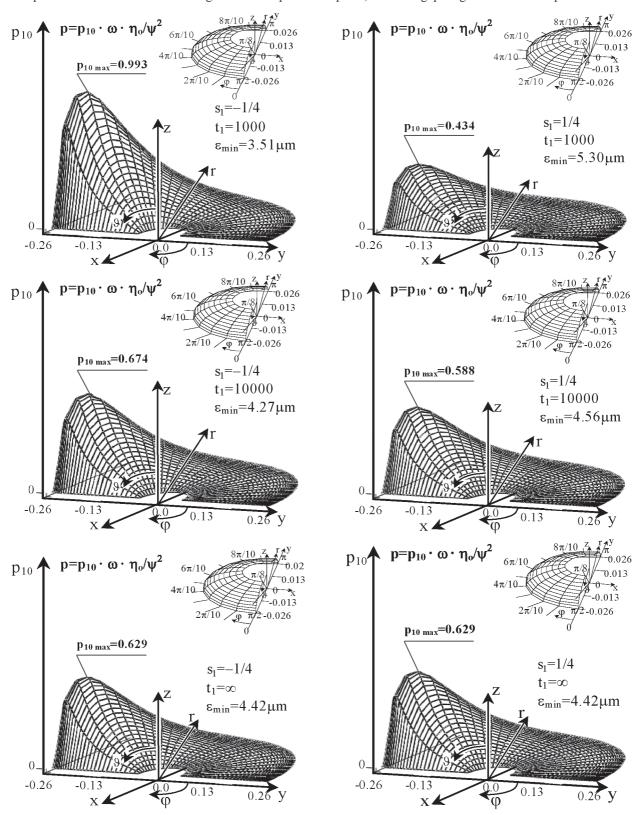


Fig.5 The dimensionless hydrodynamic pressure distributions inside the gap of slide spherical bearing, over the region $\Omega: 0 \le \varphi \le \pi$, $\pi R/8 \le \vartheta \le \pi R/2$, at the dimensionless time values: $t_1 = 1000$, $t_1 = 1000$, $t_1 \to \infty$, up to the impulse occurrence, for the increasing (decreasing) effects of the gap height changes, shown in the right (left) hand side column of the diagrams, respectively.

If the time interval up to the impulse occurrence is sufficiently large i.e. for $t_1 \to \infty$, then the pressure distributions for the increasing $(s_1 > 0)$ and decreasing $(s_1 < 0)$ effects of the gap height changes caused by the impulse, tend to the identical pressure distributions (Fig.5). Such limit pressure distribution can be also obtained from the classical Reynolds equation (52).

For the dimensionless time values : $t_1 = 1$, $t_1 = 10$, $t_1 = 100$, $t_1 = 1000$, $t_1 = 10000$, $t_1 = \infty$, i.e. for : t = 0.001s, t = 0.010 s, t = 0.100 s, t = 1.000 s, t = 0.000 s, $t = \infty$ s, after impulse occurrence the maximum pressure distributions for $t_1 < 0$ have the following dimensional values, respectively :

$$1.117 \frac{\omega \eta_{o}}{\psi^{2}} , 1.115 \frac{\omega \eta_{o}}{\psi^{2}} , 1.102 \frac{\omega \eta_{o}}{\psi^{2}} , 0.993 \frac{\omega \eta_{o}}{\psi^{2}} , 0.674 \frac{\omega \eta_{o}}{\psi^{2}} , 0.629 \frac{\omega \eta_{o}}{\psi^{2}}$$
 (66)

For the dimensionless time values : $t_1 = 1$, $t_1 = 10$, $t_1 = 100$, $t_1 = 1000$, $t_1 = 10000$, $t_1 = \infty$, i.e. for : t = 0.001s, t = 0.010s, t = 0.100s, t = 1.000s, t = 10.000s, $t = \infty s$, after impulse occurrence the maximum pressure distributions for t = 0.001s, the following dimensional values, respectively:

r

 t_1

$$0.403 \frac{\omega \eta_o}{\psi^2} , 0.403 \frac{\omega \eta_o}{\psi^2} , 0.406 \frac{\omega \eta_o}{\psi^2} , 0.434 \frac{\omega \eta_o}{\psi^2} , 0.588 \frac{\omega \eta_o}{\psi^2} , 0.629 \frac{\omega \eta_o}{\psi^2}$$
 (67)

CONCLUSIONS

- The pressure distribution changes at the instant of impulse occurrence are caused mainly by the bearing gap height changes and viscoelastic oil properties.
- ◆ The gap height changes during impulsive motion and the viscoelastic oil properties may either increase or decrease the pressure distribution and load-carrying capacity of spherical bearings in contrast to those of the same bearing but free from impulse effects.
- ◆ The pressure distribution changes and load-carrying capacity values at the instant of impulse occurrence attain about 40 percent of those appearing in the spherical bearing free from impulse effects.
- ◆ The pressure distribution changes caused by the viscoelastic oil properties can attain only about 10 percent of those appearing in the instant of impulse occurrence.
- ◆ Just after the impulse occurrence the influences of the viscoelastic oil properties on the pressure and capacity changes quickly tend to zero. This is the moment in which the largest values of wear may be expected.

Acknowledgement

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NOMENCLATURE

- acceleration vector [m/s]
- A_1 strain tensor, [1/s]
- A₂ dimensional tensor, [s⁻²]
- b_{m1},b_{s1} dimensionless origin and end coordinate of lubrication surface in meridianal direction
- c₁ dimensionless end coordinate of lubrication surface in circumferential direction
- C_{i1}, C_{i2}, C_{i3}, C_{i4} integral constant
- D dimensional eccentricity, [m]
- De Deborah Number
- Des = DeStr Deborah and Strouhal numbers
- erf special integral function
- h gap height, [m]
- h_o dimensional gap height for spherical journal
- and spherical sleeve, [m]
 dimensionless gap height
- dimensionless gap neight
- h_{min} average gap height minimum, [m] $M = Nh_1$ time depended dimensionless function
- N time depended dimensionless function
- O centre of the journal
- O_s centre of the sleeve
- p dimensional pressure, [Pa]
- p₁ dimensionless pressure
- p_{10} dimensionless pressure for Newtonian (classical) unsteady oil flow
- p₁₁, p₁₂,... dimensionless pressure corrections caused by viscoelastic oil properties in unsteady flow

- dimensional radial coordinate, [m]
- r₁ dimensionless radial coordinate
- R radius of the journal, [m]
- Re Reynolds Number
- s₁ dimensionless coefficient of gap height changes caused by the impulse
- S stress tensor, [Pa]
- Str Strouhal Number
 - dimensional time, [s]
 - dimensionless time
- characteristic value of the dimensional time, [s]
- U peripheral velocity of spherical journal, [m/s]
- v_r dimensional oil velocity component in radial
 - direction, [m/s]
- v_{r1} total dimensionless oil velocity component in radial direction
- $v_{r0\Sigma}$ dimensionless oil velocity component in radial direction for Newtonian (classical) unsteady oil flow
- $v_{r1\Sigma}, v_{r2\Sigma},...$ dimensionless corrections of oil velocity components in radial direction caused by the viscoelastic oil properties in unsteady flow
- dimensional oil velocity component in meridianal direction, [m/s]
- $v_{\vartheta 1}$ total dimensionless oil velocity component in meridianal direction
- $v_{\vartheta0\Sigma}$ dimensionless oil velocity component in meridianal direction for Newtonian (classical) unsteady oil flow
- $v_{\vartheta1\Sigma}, v_{\vartheta2\Sigma},...$ dimensionless corrections of oil velocity components in meridianal direction caused by the viscoelastic oil properties in unsteady flow
- v_{ϕ} dimensional oil velocity component in circumferential direction, [m/s]
- $v_{\phi 1}$ total dimensionless oil velocity component in circumferential direction
- $v_{\phi0\Sigma}$ dimensionless oil velocity component in circumferential direction for Newtonian (classical)
- unsteady oil flow $v_{\phi 1\Sigma}, v_{\phi 2\Sigma},...$ dimensionless corrections of oil velocity components in circumferential direction caused by the viscoelastic
- $v_{\phi1\Sigma}, v_{\phi2\Sigma},...$ -dimensionless corrections of oil velocity components in circumferential direction caused by the viscoelastic oil properties in unsteady flow
- Y,Y_1,Y_2 dimensionless functions
 - Z_1, Z_2 dimensionless functions
- α pseudo viscosity coefficient, [Pas²]
- β pseudo viscosity coefficient, [Pas²]
 - radial clearance, [m]
- $\Delta \epsilon_1$, $\Delta \epsilon_2$, $\Delta \epsilon_3$ -components of the sleeve centre, [m]
- η₀ dynamic viscosity of the oil, [Pas]
- θ meridianal direction

ω

- φ circumferential direction
- time depended dimensionless variable
- Ψ dimensionless radial clearance
 - angular velocity of the journal, [1/s]
- ω_o angular velocity of the impulsive changes caused by the perturbations in unsteady conditions, [1/s]
- lubrication surface, [m²]

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From 25 to 28 May 2004 the yearly 21st Symposium on Hydroacoustics

was held in Jurata, a touristic resort at Hel Peninsula. It was organized under the auspices of the Acoustics Committee, Polish Academy of Sciences, and the Polish Acoustical Society. The symposium was hosted by Naval University of Gdynia and Gdańsk University of Technology.

The symposia in question have been aimed at providing an opportunity for direct exchange of experience and information among teams dealing with hydroacoustics and related subjects.

> The scope of the 21st Symposium covered the following items:

- acoustic wave propagation in sea water
- hydroacoustic noise
- > non-linear acoustics in water environment
- > ultrasonic transducers
- signal processing
- hydroacoustic devices and systems
- > other related problems.

Presentation of 31 papers was performed during four plenary sessions and four topical sessions. The following papers were presented during the plenary sessions:

★ Golay's codes sequences in ultrasonography – by A. Nowicki, I. Trots, W. Secomski, J. Litniewski (Institute of Fundamental Technological Research, Polish Academy of Scieces, Warszawa)

- ★ Acoustic reconnaisance of fish and environmental background in demersal zone in Southern Baltic - by A. Orłowski (Sea Fisheries Institute, Gdynia)
- Underwater ship passport by I. Gloza (Naval University of Gdynia)
- Stability of mechanical and dielectric parameters in pzt based ceramics - by J. Ilczuk, J. Bluszcz, R. Zachariasz (University of Silesia, Sosnowiec)
- ★ *Directional sonobuoy system for detection of submarines* by R. Salamon (Gdańsk University of Technology).

Most of the presented papers were prepared by 47 authors representing 10 Polish universities and scientific centres, including the following: of Gdańsk University of Technology - 8 papers, of institutes of Polish Academy of Sciences - 6 papers, of Naval University of Gdynia - 5 papers. A group of foreign authors consisted of: 3 authors of Sevchenko Research Institute of Applied Physical Problems, Minsk, Belarus, 1 author of University of Victoria, Canada, 1 author of Institute of Applied Physics of Nizhny Novogorod, and 2 authors of Nizhny Novogorod State University, Russia, 1 author of Institute of Marine Sciences of Mersin, Turkey.



Some aspects of vibration control

Part I: Active and passive correction

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ABSTRACT



The paper presents a general approach to mechanical system modification aimed at controlling the steady harmonic vibrations by means of passive and active methods. The relative decrease of harmonic vibration amplitudes of selected elements of the mechanical system has been chosen as a measure of the quality of the introduced modification. The proposed theoretical method enables to determine the parameters of the system's dynamic flexibility matrix, which show the most remarkable effect on the dynamic behaviour of the whole system. When active control is considered the method is useful in designing the

structure and choosing the parameters of the control system. In certain cases of self-excited vibration the approach helps examining the elements of the system, most responsible for this kind of excitation.

Key words: harmonic vibrations, passive control, active control

PROBLEM DESCRIPTION

Theoretical investigations into the problem of active control of mechanical vibrations have been carried out for many years, but real-life mechanical systems making use of this idea are still rare. However recently the progress in control methods and technology has had an impact on the development of novel practical solutions, used to reduce vibration of turbomachinery rotor systems. Application of magnetic and pressurized bearings reveals new possibilities for the control of rotor behaviour. With the advent of nanotechnology the practical application of active control methods may increase in the future. The application of distributed sensor and actuator systems is tightly linked with the theory of multidimensional system control.

Let us consider a mechanical system with n degrees of freedom, Fig.1a. The inertial elements of the system can perform translational, bending and torsional movements. The forced harmonic vibration of the system is given by the matrix equation:

$$\mathbf{J} \cdot \ddot{\mathbf{Q}} + \mathbf{B} \cdot \dot{\mathbf{Q}} + \mathbf{K} \cdot \mathbf{Q} = \mathbf{F}$$
where:

- ${f J}{\ }$ the matrix of the moments of inertia of the system
- **B** the matrix of the damping coefficients of the system
- K the matrix of the stiffness coefficients of the system
- F the vector of the harmonic forces or moments acting upon the inertial elements of the system
- **Q** the vector of displacement of the inertial elements of the system.

The matrices J, B, K are of $n \times n$ dimension, and the vectors F, Q - of $n \times 1$ dimension.

The forces ${\bf F}$ acting on the system are assumed to have an identical frequency ω , and amplitudes varying within the system to be described by the vector ${\bf f}$. They may therefore be expressed as a function of time t:

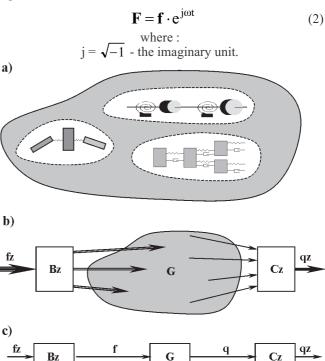


Fig 1. Schematic diagram of a mechanical system and its block diagram describing mechanical vibrations

The solution **Q** of equation (1) describing the displacement of the elements of the system may be expected to have the form:

$$\mathbf{Q} = \mathbf{q} \cdot \mathbf{e}^{\mathrm{j}\omega t} \tag{3}$$

where:

 q - the vector of displacement amplitudes of the inertial elements of the system.

After accounting for relations (2) and (3), equation (1) takes the form:

$$(-\omega^2 \mathbf{J} + \mathbf{j}\omega \mathbf{B} + \mathbf{K})\mathbf{q} = \mathbf{f}$$
 (4)

All matrices and vectors bearing the subscript i are assumed to refer to the vibrations of the mechanical system for a given frequency ω_i . For the sake of clarity it is helpful to introduce the following notation: $\mathbf{D}_i \equiv (\mathbf{K} - \omega_i^2 \mathbf{J} + j\omega_i \mathbf{B})$. The matrix \mathbf{D}_i is of n×n dimension. Equation (4) may be rewritten to include \mathbf{D}_i :

$$\mathbf{D}_{i} \cdot \mathbf{q}_{i} = \mathbf{f}_{i} \tag{5}$$

or equivalently (assuming the equation has a solution):

$$\mathbf{q}_{i} = \mathbf{G}_{i} \cdot \mathbf{f}_{i} \tag{6}$$

where

the matrix $\mathbf{G}_i \equiv \mathbf{D}_i^{-1}$ stands for the dynamic flexibility matrix of the system for the frequency ω_i .

The external forces $\mathbf{fz_i}$ are assumed to act on selected inertial elements in a way which can be described by the binary matrix $\mathbf{Bz_i}$. The matrix $\mathbf{qz_i}$ in turn represents the vibration amplitudes of the elements whose behaviour has to be controlled. The selection of these amplitudes is performed by means of the binary matrix $\mathbf{Cz_i}$. This is shown (for any vibration frequency) in Fig.1b and Fig.1c.

Active control

The behaviour of the system shown in Fig.1c is controlled by adding a feedback loop. The schematic block diagram of the active control of mechanical vibrations is presented in Fig.2.

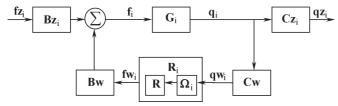


Fig 2. Block diagram of active control of mechanical vibrations

It is here assumed that only the amplitudes $\mathbf{q}\mathbf{w}_i$ of a certain number s of inertial elements (selected by the s×n binary matrix $\mathbf{C}\mathbf{w}$) can be measured. They are treated as an input value for the controller. The output vector $\mathbf{f}\mathbf{w}_i$ of the controller consists of r elements and is a function of the vector $\mathbf{q}\mathbf{w}_i$ of measured displacement, as well as of the derivative and calculus (of any order) of the displacement. The transfer matrix \mathbf{R}_i of the controller action is a r×s matrix with complex elements, which fulfils the relation:

$$\mathbf{f}\mathbf{w}_{i} = \mathbf{R}_{i} \cdot \mathbf{q}\mathbf{w}_{i} \tag{7}$$

In the case when a PID controller is used in the system, the matrix \mathbf{R}_i takes the following form :

$$\mathbf{R}_{i} = \mathbf{K}_{P} + \mathbf{j} \cdot (\mathbf{K}_{D} \boldsymbol{\omega}_{i} - \mathbf{K}_{I} \boldsymbol{\omega}_{i}^{-1})$$
 (8)

where:

 \mathbf{K}_{p} , \mathbf{K}_{I} and \mathbf{K}_{D} - r×s matrices of the proportional (P), integrating (I) and differentiating (D) controller, respectively.

Let us define the $r\times 3s$ matrix **R** of the controller parameters as :

$$\mathbf{R} = [\mathbf{K}_{P} | \mathbf{K}_{I} | \mathbf{K}_{D}]$$

The relation between matrices R_i and R may be written in the form :

$$\mathbf{R}_{i} = \mathbf{R} \cdot \mathbf{\mathring{U}}_{i}$$
 where:

$$\mathbf{\mathring{U}}_{i} = \begin{bmatrix} -\frac{\mathbf{I}_{s}}{-j\omega_{i}^{-1} \cdot \mathbf{I}_{s}} \\ -\frac{j\omega_{i}}{j\omega_{i} \cdot \mathbf{I}_{s}} \end{bmatrix}$$

When taking into consideration controller models with differentiation or integration of a higher order it is sufficient to extend the parameter matrix \mathbf{R} horizontally, and appropriately extend the frequency multiplier matrix Ω vertically.

The steering force signal $\mathbf{fw_i}$ is passed onto the active elements of the control feedback loop. The locations of the elements upon which they act are given by the n×r matrix \mathbf{Bw} .

The aim of the controller R is to minimize the vibration amplitudes of selected elements described by the matrix qz_i .

Passive control

Passive control is here understood as the modification of parameters of the mechanical system. This may be achieved by introducing the changes $\mathbf{P} = [\Delta \mathbf{J} \mid \Delta \mathbf{B} \mid \Delta \mathbf{K}]$ to certain system parameters (i.e. inertia, damping or stiffness coefficients), selected by the binary matrices $\mathbf{B}\mathbf{w}$ and $\mathbf{C}\mathbf{w}$. For a given frequency ω_i the changes introduced to the matrix \mathbf{D}_i lead to the following changes in equation (5):

$$(\mathbf{D}_{i} + \mathbf{B}\mathbf{w} \cdot \mathbf{P}_{i} \cdot \mathbf{C}\mathbf{w})\mathbf{q}_{i} = \mathbf{f}_{i}$$
where (9)

$$P_i = P \Omega_i$$

From equation (9) one obtains:

$$\mathbf{q}_{i} = \mathbf{D}_{i}^{-1} \mathbf{f}_{i} - \mathbf{D}_{i}^{-1} \mathbf{B} \mathbf{w} \cdot \mathbf{P}_{i} \cdot \mathbf{C} \mathbf{w} \cdot \mathbf{q}_{i}$$
 (10)

By taking into account the notation: $\mathbf{G}_{i} \equiv \mathbf{D}_{i}^{-1}$, equation (10) may be written in the form:

$$\mathbf{q}_{i} = \mathbf{G} \cdot \mathbf{f}_{i} - \mathbf{G} \cdot \mathbf{B} \mathbf{w} \cdot \mathbf{P}_{i} \cdot \mathbf{C} \mathbf{w} \cdot \mathbf{q}_{i}$$
 (11)

Relation (11) is presented in the form of the block diagram shown in Fig 3.

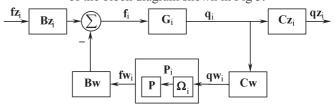


Fig 3. Block diagram of harmonic vibrations with correction of system parameters

The aim of selecting the matrix P_i (which describes changes of system parameters) is the minimization of the vibration amplitudes of chosen elements described by the matrix qz_i . It should be emphasized that harmonic vibrations with the correction of system parameters and the process of active control of mechanical vibrations may by represented by the same general form of block diagram (compare Fig.2 and 3).

Some cases of self-excited vibrations

In many cases the so-called self-excited vibrations depend on the behaviour of the mechanical system itself. Let us consider, as an example, self excited vibrations of a turbomachinery

rotor system due to aerodynamic forces. Rotor-stator eccentricity or rotor-stator misalignment changes the clearance distribution above the blade shroud (and in glands) which results in aerodynamic forces and moments acting on the turbine rotor. As a result, self-excited vibrations of the whole rotor system may be observed. Due to inaccuracy of manufacture and assembly, the value and the distribution of the clearance in particular seals may differ significantly, and seals of different types can be used in the same machine, thus the aerodynamic forces acting on a rotor in each stage can vary remarkably. Various theoretical models have been elaborated to describe the fluid motion in the seals and to determine the aerodynamic forces generated in a shroud clearance. Usually the aerodynamic forces and moments are expressed in the form of rotodynamic coefficients [1÷ 6, 8, 9, 12, 13] which can be written in the form of the following vector equation:

$$\mathbf{F} = \mathbf{J}_{\mathbf{u}}\ddot{\mathbf{Q}} + \mathbf{B}_{\mathbf{u}}\dot{\mathbf{Q}} + \mathbf{K}_{\mathbf{u}}\mathbf{Q} \tag{12}$$

where

- F vector of the components of the aerodynamic forces and moments
- Q rotor displacement and rotation in horizontal and vertical directions

 J_u , B_u and K_u - matrices with the so called "inertia", "damping" and "stiffness" coefficients of the shroud (or gland), respectively.

The symbol **S** is used to denote the matrix of rotordynamic coefficients : $\mathbf{S} = [\mathbf{J}_{\mathbf{u}} | \mathbf{B}_{\mathbf{u}} | \mathbf{K}_{\mathbf{u}}]$.

Because the self-excited vibrations occur at a certain frequency ω_i , equation (12) takes a form similar to that of equation (4):

$$\mathbf{B}\mathbf{w}\left(-\omega_{i}^{2}\mathbf{J}_{n}+\mathrm{j}\omega_{i}\mathbf{B}_{n}+\mathbf{K}_{n}\right)\mathbf{C}\mathbf{w}\cdot\mathbf{q}_{i}=\mathbf{f}_{i} \qquad (13)$$

The matrix **Cw** selects the displacements responsible for aerodynamic excitations, while the matrix **Bw** determines the places where these forces are applied.

The equation (13) can also be written in the form:

$$\mathbf{B}\mathbf{w} \cdot \mathbf{S}_{i} \cdot \mathbf{C}\mathbf{w} \cdot \mathbf{q}_{i} = \mathbf{f}_{i}$$

where :

$$\mathbf{S}_{i} \equiv -\omega_{i}^{2} \mathbf{J}_{u} + j\omega_{i} \mathbf{B}_{u} + \mathbf{K}_{u} = \mathbf{S} \cdot \mathbf{\Omega}_{i}$$

When taking into consideration only the aerodynamic excitation forces, the behaviour of the rotor system can be represented by means of the block diagram shown in Fig.4.

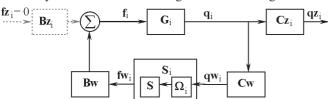


Fig 4. Block diagram of harmonic self-excited vibrations

The comparison of Fig.2 , 3 and 4 leads to the conclusion that harmonic vibrations with the correction of system parameters, the process of active control of mechanical vibrations as well as some cases of self-excited vibrations can by represented by one and the same general form of block diagram. Thus all the three cases may be generalized to the form presented in Fig.5. The matrix \mathbf{U}_i can represent any of the matrices $\mathbf{R}_i, \, \mathbf{P}_i, \, \mathbf{S}_i,$ depending on the context. All further considerations are conducted with the use of this general form, and the attention is focused on determining the value of the coefficients of the matrix \mathbf{U} for which the amplitudes qz of chosen elements of the system (described by the dynamic flexibility matrix \mathbf{G}) will be minimum.

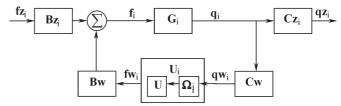


Fig 5. Block diagram of the system performing mechanical vibrations

When taking into consideration vibrations of a mechanical system with k different values of frequency ω it is possible to generalize the model given in Fig.5 to the form shown in Fig.6. This is performed by introducing generalized matrices G, Ω , \widetilde{U} , \overline{U} , \widetilde{B} w, \widetilde{C} w, Bz, Cz, and vectors f, fz, fw, q, qz, qw (Fig.6), which correspond, respectively, to the matrices G_i , Ω_i , U, U_i , Bw, Cw, Bz_i , Cz_i , and vectors f_i , fz_i , fw_i , q_i , qz_i , qw_i in the case of vibrations with a single frequency ω_i (Fig. 5).

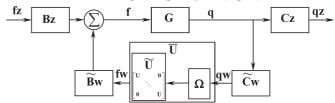


Fig 6. Block diagram of active control of mechanical vibrations

All of the vectors \mathbf{f} , \mathbf{fz} , \mathbf{fw} , \mathbf{q} , \mathbf{qz} , \mathbf{qw} in the generalized scheme are column vectors composed of the corresponding vectors for a single frequency. In other words, these vectors may be written in the form :

$$\mathbf{f} = \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \vdots \\ \mathbf{f}_k \end{bmatrix} \quad \mathbf{fz} = \begin{bmatrix} \mathbf{fz}_1 \\ \mathbf{fz}_2 \\ \vdots \\ \mathbf{fz}_k \end{bmatrix} \quad \mathbf{fw} = \begin{bmatrix} \mathbf{fw}_1 \\ \mathbf{fw}_2 \\ \vdots \\ \mathbf{fw}_k \end{bmatrix}$$

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \vdots \\ \mathbf{q}_k \end{bmatrix} \quad \mathbf{qz} = \begin{bmatrix} \mathbf{qz}_1 \\ \mathbf{qz}_2 \\ \vdots \\ \mathbf{qz}_k \end{bmatrix} \quad \mathbf{qw} = \begin{bmatrix} \mathbf{qw}_1 \\ \mathbf{qw}_2 \\ \vdots \\ \mathbf{qw}_k \end{bmatrix}$$

The matrices G, Ω , \widetilde{U} , \overline{U} , $\widetilde{B}w$, $\widetilde{C}w$, Bz, Cz are described by the following formulas :

$$\begin{split} \mathbf{G} &= \begin{bmatrix} \mathbf{G}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{G}_k \end{bmatrix} \mathbf{\Omega} = \begin{bmatrix} \mathbf{\Omega}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Omega}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{\Omega}_k \end{bmatrix} \\ \widetilde{\mathbf{U}} &= \begin{bmatrix} \mathbf{U} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{U} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{U} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{U} \end{bmatrix} \mathbf{\overline{U}} = \begin{bmatrix} \mathbf{U}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{U}_k \end{bmatrix} \\ \widetilde{\mathbf{B}} \mathbf{w} &= \begin{bmatrix} \mathbf{B} \mathbf{w} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \mathbf{w} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{B} \mathbf{w} \end{bmatrix} \mathbf{C} \mathbf{w} = \begin{bmatrix} \mathbf{C} \mathbf{w} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{C} \mathbf{w} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{C} \mathbf{w} \end{bmatrix} \\ \mathbf{B} \mathbf{z} &= \begin{bmatrix} \mathbf{B} \mathbf{z}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \mathbf{z}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{B} \mathbf{z}_k \end{bmatrix} \mathbf{C} \mathbf{z} = \begin{bmatrix} \mathbf{C} \mathbf{z}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{C} \mathbf{z}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{C} \mathbf{z}_k \end{bmatrix} \end{aligned}$$

It is useful to note that all the generalized matrices may be described as a linear combination of the corresponding matrices for particular frequencies and certain other fixed matrices. In particular, for the matrix $\tilde{\mathbf{U}}$ it can be written:

$$\widetilde{\mathbf{U}} = \sum_{i=1}^{k} \left[(\mathbf{e}_{i} \times \mathbf{I}_{s}) \cdot \mathbf{U} \cdot (\mathbf{e}_{i}^{T} \times \mathbf{I}_{3r}) \right]$$
(14)

where

 e_i - the i-th versor of dimension k, the symbol : \times stands for the carthesian product of matrices.

METHOD OF RESPONSE CIRCLES

The following set of equations can be written for the system presented in Fig.6:

$$\mathbf{q} = \mathbf{G} \cdot \mathbf{f} \tag{15}$$

$$\mathbf{f} = \widetilde{\mathbf{B}}\mathbf{z} \cdot \mathbf{f}\mathbf{z} + \mathbf{B}\mathbf{w} \cdot \mathbf{f}\mathbf{w} \tag{16}$$

$$\mathbf{q}\mathbf{z} = \widetilde{\mathbf{C}}\mathbf{z} \cdot \mathbf{q} \tag{17}$$

$$\mathbf{q}\mathbf{w} = \mathbf{C}\mathbf{w} \cdot \mathbf{q} \tag{18}$$

$$\mathbf{fw} = \overline{\mathbf{U}} \cdot \mathbf{qw} \tag{19}$$

$$\overline{\mathbf{U}} = \widetilde{\mathbf{U}} \cdot \mathbf{\Omega} \tag{20}$$

By combining equations $(14) \div (22)$ the following relation can be derived:

$$\mathbf{qz} = \mathbf{Cz} \left(\mathbf{I}_{n} - \mathbf{G} \cdot \widetilde{\mathbf{B}} \mathbf{w} \sum_{i=1}^{k} \begin{bmatrix} (\mathbf{e}_{i} \times \mathbf{I}_{s}) \cdot \\ \mathbf{U} \cdot (\mathbf{e}_{i}^{\mathsf{T}} \times \mathbf{I}_{3r}) \end{bmatrix} \mathbf{\Omega} \cdot \widetilde{\mathbf{C}} \mathbf{w} \right)^{-1} \cdot \mathbf{G} \cdot \mathbf{fz}$$
(21)

In the case without any feedback (U=0) the vector qz_0 of amplitudes of selected elements may be written in the form :

$$\mathbf{q}\mathbf{z}_0 = \mathbf{C}\mathbf{z} \cdot \mathbf{G} \cdot \mathbf{f}\mathbf{z} \tag{22}$$

By using equations $(15) \div (21)$ it is possible to investigate how the real or imaginary part u of a particular coefficient of the matrix \mathbf{U} influences the amplitudes in vector \mathbf{qz} . The vibrations of particular elements represented by the matrix \mathbf{qz} can play a different role in the dynamic behaviour of the mechanical system. Therefore in some cases it is useful to use a weighted sum of amplitudes as a measure of the vibration level:

$$\sigma_{z} = \sum_{i} \mathbf{a}[i] \cdot \mathbf{q} \mathbf{z}[i]$$
 (23)

where:

a[i] - the weight coefficient corresponding to amplitude qz[i].

For the case of no feedback it can be similarly written:

$$\sigma_{z0} = \sum_{i} \mathbf{a}[i] \cdot \mathbf{q} \mathbf{z}_{0}[i]$$
 (24)

After some transformation it is possible to show that the ratio ζ of the indexes σ_z and σ_{z0} may be written in the general form :

$$\zeta(u) = \frac{\sigma_z(u)}{\sigma_{z0}(u)} = 1 + \sum_{i=1}^k \frac{u(a_i + jb_i)}{1 - u(c_i + jd_i)}$$
(25)

where

 a_i , b_i , c_i , d_i , $(1 \le i \le k)$ are real numbers.

The module $|\zeta|$ shows the effect of the feedback loop on the weighted vibration amplitude of the elements of qz. In general, when $\forall_{1 \le i \le k} [d_i \ne 0 \land (a_i \ne 0 \lor b_i \ne 0)]$ equation (25) represents a closed smooth curve which is described by the end of the vector ζ in the complex coordinate system (r, i) when u

varies from $-\infty$ to $+\infty$. An example of such curve is presented for k=4 in Fig.7.

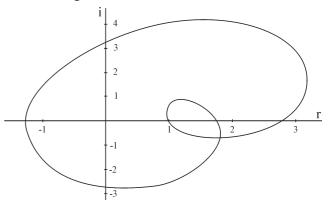


Fig 7. Graphical interpretation of equation (25) with exemplary coefficients for k different frequencies (k = 4)

When taking into consideration only a particular frequency ω , equation (25) may be written in the form:

$$\zeta(u) = 1 + \frac{u(a+jb)}{1 - u(c+jd)}$$
 (26)

where

a, b, c, d, are real numbers.

In the usual case when $d \neq 0 \land (a \neq 0 \lor b \neq 0)$, equation (26) represents a circle which is described by the end of the vector ζ in the complex coordinate system (r, i) when u varies from $-\infty$ to $+\infty$ [7]:

$$\left[r - \left(1 - \frac{b}{2d}\right)\right]^{2} + \left[i - \frac{a}{2d}\right]^{2} = \frac{a^{2} + b^{2}}{4d^{2}}$$
 (27)

A graphical interpretation of equations (26, 27) is shown in Fig.8.

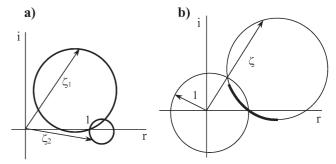


Fig 8. Graphical interpretation of equation (27)

The diameter of the closed curve (in particular – a circle) described by the vector ζ enables to assess the influence of the parameter u on the weighted amplitude of vibrations. The large diameter of the curve (the circle described by the vector ζ_1 in Fig.8a) may be interpreted as a significant influence of the parameter u, while the small diameter (the circle determined by ζ_2 in Fig.8a) – as an insignificant influence of the parameter u. However in practice the parameter u has a reasonably limited range of values. Thus only a part of the curve drawn by the vector ζ can be applied in practice (for example only the marked part of the circle shown in Fig. 8b). Moreover, only the values of parameter u, for which the module $|\zeta|$ is lesser than 1, result in the decrease of the weighted amplitude level. In this way it is possible to estimate the effect of all parameters u on the vibrations and thus to choose :

* the controller structure and its parameters for the best active control of mechanical vibrations

- the changes of system parameters leading to the most effective reduction of vibrations
- * the shrouds and the glands which play the most important part in generating forces and moments in the case when rotor self-excited vibrations of aerodynamic type are considered.

EXAMPLES

A ship propulsion system

Ship propulsion systems equipped with flexible couplings are very sensitive to disturbances caused by unsteady engine operation. The disturbances have the form of shaft torque periodical changes which lead to torsional vibrations of the whole propulsion system. In some cases resonance vibrations resulting in damages to flexible couplings, were observed. This situation very often occurs when the engine works with one misfiring cylinder. In Fig.9a an example ship propulsion system is presented.

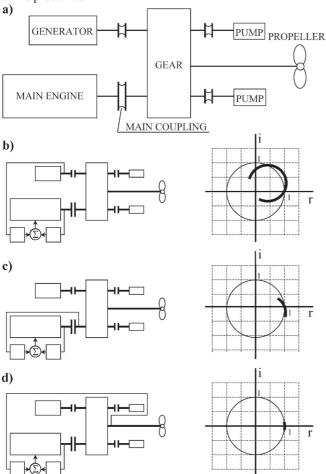


Fig 9. Variants of additional control systems and their response circles

It consists of a main medium-speed diesel engine which — through a main coupling and a mechanical gear — drives a ship propeller, an electric generator and two hydraulic pumps. All the couplings are flexible. The linear model of this system and the analysis of its behaviour was elaborated [11] by using the engine producer's data and results of some additional investigations. The reduction of the main coupling torsional vibration was performed by modifying the main engine governor. Three system variants were considered for the following different correction input signals:

- > angular velocity of the generator (Fig.9b)
- > angular velocity of the main coupling (before the gear) (Fig.9c)

angular velocity of the propeller shaft (measured directly after the gear) (Fig.9d).

In all three cases the presented method of response circles was applied to the analysis of the structure and parameters of the additional correction controller. The reduction ratio ζ of the main coupling vibration amplitude for the case of a proportional controller, 270 rpm shaft speed and 14 Hz fundamental harmonic frequency of forced vibrations, is shown in Fig.9b \div 9d for the above mentioned system variants, respectively.

From Fig.9 it is evident that the controller using the generator's angular velocity as its correction signal offers the largest possibilities of reducing torsional vibration amplitude in the main coupling.

Turbine rotor self-excited vibrations

The method of response circles was used to select the seals which play the most important part in generating aerodynamic forces leading to self-excited vibrations of the rotor system. The forces were described by means of the rotordynamic coefficient matrix $\mathbf{S} = [\mathbf{J}_u \mid \mathbf{B}_u \mid \mathbf{K}_u]$, and calculated from relation (13). The relevant schematic diagram is shown in Fig.10.

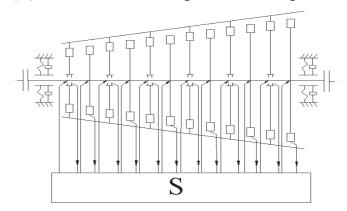


Fig 10. Schematic diagram for rotodynamic coefficient analysis

In the case of a double-cylinder medium-power steam turbine the performed analysis proved that the seals of the first stages of the HP cylinder had the greatest influence on self-excited vibrations of the aerodynamic type. It was enough to change the seals of the shrouds and shaft in first four turbine stages to achieve the desired effect of vibration reduction.

A currently conducted work is concentrated on active control of rotor vibrations of a steam turbine by means of pressurized bearings. The method of response circles is used to detect the bearings which have the greatest influence on active control. Results of the work in question will be presented in a separate paper in due course.

CONCLUSIONS

- O An analytical method for the investigation of linear mechanical systems performing harmonic motion was presented. This approach was successfully applied for the analysis and improvement of the dynamic behaviour of a ship propulsion system.
- O The proposed theoretical method makes it possible to determine the parameters of the system's dynamic flexibility matrix which show the most remarkable effect on the dynamic behaviour of the whole system.
- O When active control is considered the method is useful in the designing of the structure and choice of parameters of the control system.
- O In certain cases of self-excited vibrations the approach helps examining the elements of the system which are most responsible for this kind of excitation.

In Part II of the paper (to be published) a theoretical method of optimum vibration control by active means is described and illustrated by some examples, including an approach to pressurized bearing application.

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The author wishes to express his gratitude to Prof. Włodzimierz Gawroński (NASA - National Aeronautics & Space Agency) who many years ago has encouraged investigations into the subject of active control of mechanical vibrations. Both parts of this paper constitute the generalized approach to the problem which was elaborated in close cooperation with Prof. Gawroński [10].

NOMENCLATURE

B - matrix of damping coefficients
C - selection matrix

Bw, Bz, Cw, Cz - binary matrices
transfer matrix

f - vector of force amplitudes fw - controller output vector fz - vector of external forces

F - vector of harmonic forces (or moments)

G - dynamic flexibility matrix

I - unitary matrix j - imaginary unit

J - matrix of inertia moments
k, n, r, s - dimensions of matrices and vectors
K - matrix of stiffness coefficients
P - matrix of changes of system parameters

q - vector of displacement amplitudes qw - vector of measured amplitudes

qz - vector of amplitudes of controlled elements

Q - vector of displacements
R - controller matrix

S - matrix of rotordynamic coefficients J_n , B_n , K_n

t - time

U - general symbol for the matrices P, R, S

σ - weighted sum of amplitudes

 ω - frequency

Ω - frequency multiplier matrix

Indices: P - proportional controller

I - integrating controllerD - differentiating controller

u - of rotordynamic coefficients of turbine seals

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Miscellanea

10th Anniversary of Polish Maritime Research

10 years have already gone by since the first issue of the scientific technical journal Polish Maritime Research was published. During this time 42 issues of the journal, which contained 210 papers presenting scientific achievements of 190 authors, appeared. High scientific quality of the publications has been guaranteed by the Editorial Board members as well as 72 reviewers of recognized prestige in worldwide scientific circles. The journal has been published under the auspices of the Society of Polish Naval Architects and Marine Engineers, Polish Safety and Reliability Association and three special scientific bodies of the Polish Academy of Sciences.

The journal is presented in Internet under the address http://www.bg.pg.gda.pl/pmr.html
as well as it is represented in the worldwide web bibliographical databases:

☆ INSPEC (English)☆ VINITI (Russian)

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On possible supplying ship diesel engines with alternative fuels (mixtures of diesel oils and vegetable oils or their esters)

Preliminary report

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ABSTRACT

The paper presents introduction to the research on possible supplying ship diesel engines with mixtures of diesel oils and vegetable oils or their esters with accounting for ecological aspects, i.e. exahust gas purity. Characterisitics of vegetable oils and their esters are compared with those of diesel oils; some consequences of their application to diesel engines, mainly for their working process and exhaust gas content, are indicated. Also, influence of combusting their mixtures with diesel oils are discussed in the same context. Scope of the planned research project is shortly presented.

Key words: ship diesel engines, alternative fuel oils, ecology

INTRODUCTION

Contemporary main diesel engines of sea-going ships are commonly supplied with heavy oil fuels. This very often concerns also auxiliary engines, especially electric generating sets.

However on many ships the electric generating sets are still fed with marine diesel oils (MDO). Also, most of diesel engines installed on small ships are run on MDO.

Permanently increasing demand of diesel oils (**DO**), increase of their prices, increasing ecological requirements make that more and more attention is paid to the alternative fuels called also substitute, renewable or unconventional. Another reason of the growing interest to the fuels is the increasing probability of dropping worldwide output of crude oil due to different causes. Moreover the problem of the hazard to natural environment, associated with mining, transport, processing and combusting the oil products is today brought up more and more strongly.

All energy sources other than crude oil products may be deemed unconventional ones [1]. As it results from the diagram presented in Fig.1. there is a wide range of possible media for supplying diesel engines. However today most of the unconventional fuels are not used in practice of operation of diesel engines, including ship engines.

In the research project planned by these authors the supplying of a ship diesel engine only with a mixture of marine diesel oil (MDO) and rape-feed oil methyl ester (RME) has been accounted for.

Application of vegetable oils for supplying diesel engines

In the northern zone of moderate climate rape oil, flaxseed oil and corn oil are mostly taken into account. In other climatic zones it can be soybean, sunflower, palm, cotton, sesame, peanut or coconut oil.

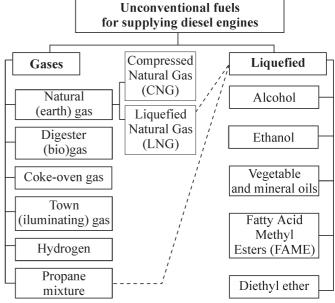


Fig. 1. Scope of unconventional fuels for supplying diesel engines [1]

Vegetable oils are esters of glycerol and fatty acids containing from 14 to 22 carbon atoms [5]. In Tab.1 [1] some properties of diesel oils and selected vagetable oils are compared.

It can be observed that some parameters of vegetable oils, their esters and diesel oils are of similar values, respectively. However their density, kinematic viscosity and flow temperature values much differ from those of diesel oils.

As far as rape oils are concerned (most interesting in case of Poland) their density and viscosity is distinctly higher, which can make supplying diesel engines with them difficult, however their positive features are: practically no sulphur content and bio-degradation ability. Their calorific value is smaller than

Tab. 1. Comparison of some properties of diesel oils, vegetable oils and methyl esters of higher fatty acids of rape oil (RME)

Parameter	Unit	Diesel oils (DO)	Vegetable	Esters	
1 ar ameter		Diesei olis (DO)	rape oil (RO) palm oil (PO		(RME)
Density at 15°C	kg/m³	820 ÷ 860	920	899	860 ÷ 900
Kinematic viscosity at:					
40°C	mm ² /s	$1.5 \div 4.5$	$30.0 \div 43.0$	39.3	$4.3 \div 6.3$
100°C		0.75	$8.0 \div 8.4$	8.4	~ 1.8
Cetane number		45 ÷ 55	~ 51	~ 51	49 ÷ 56
Gross calorific value	MJ/kg	42 ÷ 45	37.1 ÷ 37.5	37.3	37 ÷ 39
Flow temperature	°C	< -15	- 6	38	- 5 ÷ - 8
C/H/O ratio	% masy	86/14/0	77/12/11	77/12/11	-
Sulphur content	mg/kg	< 350	1	< 1	10 ÷ 25

that of diesel oils, due to different chemical content. Results of research on application of vegetable oils for supplying diesel engines [2, 3, 4, 9, 10] show worse cylinder filling, worse spraying, greater lengths of injected oil jets, associated with their large viscosity and density. Engines supplied with rape oil operate with a lower total efficiency mainly due to its lower calorific value in comparison with that of diesel oils, as well as due to worse spraying process resulting in a longer combustion. There is no unambiguous results of research on exhaust gas toxicity. Most obtained data indicate an improvement of exhaust gas purity relative to its content in the case of supplying engines with diesel oil. However the phenomena associated with supplying diesel engines with rape oil are disturbing, namely:

- ★ often occurrence of clogging sprayer nozzles
- ★ great susceptibility to forming carbon deposits on piston heads, ring grooves, valves and valve seats

According to [1,10] 20% addition of diesel oil to rape oil made its viscosity dropping by 30% as well as ignition lag period lowering; engine starting features appeared improved. In common operational applications small additions of rape oil or RME to diesel oil (e.g. 5% to 20%) are usually reported. Tests on mechanical vehicles running on a 20% RO / 80% DO mixture did not reveal any detrimental consequences [10].

Therefore these authors have decided to carry out investigations on diesel engines supplied with **MDO/RME** mixtures initially containing no more than 10% of **RME**.

The basic properties of the **MDO** and **RME** and their mixtures selected for the experiments are given in Tab.2.

Preparation of such mixture is easy as it does not reveal a tendency to separation. However such mixtures could be less ressistant to ageing process hence their should be consumed in a short time.

Tab. 2. The basic properties of the MDO and RME and their mixtures selected for the experimental tests

Oils	Density	Viscosity				Viscosity			
	kg/m ³	°E			cST (mm ² /s)				
		20°C	50°C	70°C	80°C	20°C	50°C	70°C	80°C
Fuel oil (MDO)	831	1.31	1.11	1.04	1.01	4.2	2.1	1.4	1.1
Rape oil methyl ester (RME)	883	1.79	1.29	1.13	1.12	9.3	4.0	2.3	2.2
95% MDO + + 5% RME	833	1.38	1.15	1.08	1.01	4.7	2.5	1.8	1.1
85% MDO + + 10% RME	836	1.38	1.14	1.06	1.03	4.9	2.4	1.6	1.3

- ★ troubles with starting engines at low ambient temperatures
- ★ seizing precise pairs of injection pumps.

For these reasons it seems not advisable to commonly use vegetable oils as fuels for diesel engines. However much better effects can be obtained by applying chemically modified oils, i.e. methyl esters of higher fatty acids (FAME), see Fig.1. In Poland and many other European countries rape oil and methyl alcohol are usually applied to produce the esters called rape-feed oil methyl esters (RME).

Mixtures of diesel oils and vegetable oils or their esters

Due to substantial difficulties in applying only vegetable oils as well as due to limitation in using esters for running diesel engines, an alternative is to use mixtures of diesel oils and vegetable oils or diesel oils and vegetable oil esters. Properties of such mixtures depend on properties of their components and their content in a mixture. In this way it is expected to decrease density and viscosity of a mixture relative to those of a given vegetable oil or ester.

SCOPE OF THE PLANNED RESEARCH PROJECT

The planned research project is aimed at revealing:

- + the influence of using MDO/RME mixture to running a ship diesel engine on its operation
- other possible problems arising from using such fuel mixture, e.g. its durability, storage conditions, transport, filtration processes etc.

The investigations will be carried out with the use of the L22 diesel engine installed in the laboratory of ship combustion engines, Gdynia Maritime University. The laboratory engine and test stand to be used were described in [6, 7, 8].

The planned tests will have a comparative character. The first series of all the tests will be devoted to the investigations of the engine fed with pure marine diesel oil, results of which will be taken as model ones. The consecutive test series will be carried out with MDO/RME mixtures of various content, and the controlled parameters such as engine speed or load will be changed in the same way as in the model series. Both absolute and relative values of the parameters will be analyzed.

The largest group of ship diesel engines working on diesel oil are those driving electric generating sets, which, due to their function, are of constant speed and highly changeable load (torque or fuel oil charge). Therefore the first tests will be performed for such loading mode of the engine. At different but constant engine speed values the engine load will be changed within the range from 20% to 80% of the rated torque, during which all important parameters of the engine working process as well as indication diagrams will be recorded. During the tests special attention will be paid to exhaust gas content both from the point of view of engine working process course and noxious component content.

In the first phase of the research the use of two fuels (of 5% and 10% of RME content in **MDO/RME** mixture) is assumed because in the case no changes in the engine's construction or regulation system are necessary.

The next phase of the research should be focused on durability (reliability) of the engine installed on a ship. However decision on initiating such tests will be taken on the basis of analysis of the results obtained in the first phase of the research.

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EXPLO-SHIP 2004

On 31 May and 1 June 2004 3rd International Scientific Technical Conference on :

Operational Problems of Floating Objects and Port Facilities

was held in Świnoujście, a Baltic coast port and health resort.

It was organized by Sea Navigation Institute, Maritime University of Szczecin.

During its scientific part 124 papers including the following 5 plenary ones, were presented:

- ★ Coordinate systems applied to navigation over limited water regions – by S. Gucma (Maritime University of Szczecin
- ★ Some proposals on an unified maritime safety and security system by Z. Kopacz, W. Morgaś, J. Urbański (Naval University of Gdynia)
- ★ Operational reliability of transport systems by J. Jaźwiński, Z. Smalko, J. Żurek (Air Force Institute of Technology, Warszawa)
- ★ Prospects of application of natural gas to ship engines by St. Żmudzki (Technical University of Szczecin)
- ★ Contemporary systems for determining search areas at sea by Z. Burciu, J. Soliwoda, S. Ukleja (Gdynia Maritime University)

The remaining 119 papers were presented during 8 topical sessions on :

- ★ Modelling and simulation in navigation (15 papers)
- **→** *Operation of ship* (13 papers)
- → Navigational systems (16 papers)
- → Safety at sea (12 papers)
- *★ Ship combustion engines* (14 papers)
- **→** *Ship systems* (17 papers)
- ★ State identification (16 papers)
- → Operation of ship systems (16 papers).

Several authors of the papers came from Russia and Ukraine. An attractive end of the Conference was the touristic voyage of its participants to Copenhagen, Denmark, onboard a ferryship.





PIAP - Automation 2004

On 24-26 March 2004 the Technical Scientific Conference on :

Automation - novelties and prospects

organized by the Industrial Institute for Automation and Measurements, was held in Warszawa.

It was already the 8th - since 1997 - meeting of experts from scientific, research and development centres, as well as industrial enterprises, which was a good opportunity to present achievements and to exchange experience in the area of practical applications of means for automation and robotics as well as of measurement instruments and systems applicable to various fields of engineering.

The Conference program comprised 4 plenary papers and 61 papers to be presented during six topical sessions as follows:

- I Automation, robotics, monitoring (34 papers)
- II Software, equipment and applications of mobile robots (7 papers)
- III Methods of design and integration of systems (4 papers)
- IV Devices for automation and robotics (9 papers)
- V Measurement instruments and systems (5 papers)
- VI Industrial network communication systems (2 papers).

The authors of the papers represented over 20 universities, scientific institutes and research centres, among which specialists from Czech Republic and Germany were also present. The greatest number (14) of the papers were prepared by the authors from Warsaw University of Technology, and 9 and 8 - by the authors from the Industrial Institute for Automation and Measurement, Warszawa, and Silesian University of Technology, Gliwice, respectively.

The following papers, directly or indirectly dealing with maritime engineering and economy, were presented in the plenary session:

- Methods and systems for monitoring industrial processes – by J. M. Kościelny (Warsaw University of Technology)
- ❖ Synergy in mechatronics by Z. Gosiewski (Military University of Technology, Warszawa)

Session I:

- PID or fuzzy logic control of water level in steam boiler? – by P. Biały and S. Skoczowski (Technical University of Szczecin)
- Dynamic decoupling of right-hand-side convertible dynamic systems – by P. Dworak and S. Bańka (Technical University of Szczecin)
- ❖ Steadfast MFC-PID temperature controller and its realization by means of PLC programmer – by K. Pietrusewicz and S. Skoczowski (Technical University of Szczecin

Flow control of pallets applied in a research flexible manufacturing system – by J. Honczarenko and M. Sosnowski (Technical University of Szczecin)

Session II:

- Diagnostics of failure states of an underwater vehicle in service conditions – by T. Leszczyński (Polish Naval University, Gdynia)
- ❖ Automatic control of immersion depth of an underwater vehicle by P. Szymak (Polish Naval University, Gdynia)

Session III:

Aiding of design of ship power plant automation systems by using of CBR method – by M. Meler-Kapcia, Z. Kowalski and S. Zieliński (Gdańsk University of Technology)

Session IV:

- Computer testing of automatic synchronizers for electric generators by A. Grono, G. Redlarski and J. Zawalicz (Gdańsk University of Technology)
- Conical magnetic bearings by Z. Gosiewski, K.Falkowski (Military University of Technology, Warszawa) and L. Matuszewski (Gdańsk University of Technology)

Session VI:

Application of USB buses to microprocessor automation systems – by M. Porzeziński and M. Drewka (Gdańsk Universty of Technology).

The Conference participants were also given the opportunity to visit 10th International Fair for Measurement and Control and 5th International Fair for Pumps and Industrial Fittings.



AIWARM 2004

 $On\ 26\mbox{-}27\ August\ 2004$ the Asian International Workshop on :

Advance Reliability Modelling

had place in Hiroshima City, Japan. During 20 topical sessions 79 papers were presented. Their authors came from scientific centres of Canada, China, India, Italy, Japan, Korea, Kuwait, New Zealand, Russia, Singapore, South Africa, Sweden, Taiwan, Thailand and USA.

Prof. K. Kołowrocki, a representative of Gdynia Maritime University (Poland) was among them, who presented the ordered paper on :

Reliability and risk evaluation of large systems.

CON CONCON

Science - Practice- Education

Under this heading scientific cooperation among mechanical faculties of Polish and German universities is developed. Its starting point was a symposium held in Bremen in 1989, in which representatives of local universities and scientific workers from Mechanical Faculty, Gdańsk University of Technology, took part.

With time, the yearly meetings during which experience and scientific information has been exchanged on the basis of presented papers, has stirred up greater and greater interest. Both number of participants and group of coorganizing universities has been growing and program of the symposia enriched by visits to industrial enterprises and participation in local jubilee ceremonies.

On 5÷7 May 2004 14th Symposium of the kind, organized by a German Technical University, was held in Stralsund.

From Polish side 8 persons from Gdańsk University of Technology, 4 - from Technical University of Szczecin, 3 - from the State Higher Engineering School of Elblag, and 1 - from the Fluid-Flow Machinery Institute, Polish Academy of Sciences, Gdańsk, took part in the Symposium.

The following papers were presented by Polish participants:

from Gdańsk University of Technology

- ★ Investigations of wall influence on single bubble movement by D. Mikielewicz and J.Wajs
- ★ Hydrodynamic Tunnel the test stand for investigation of characteristics of profiles and cascades by J. Iwan, B. Lednicka, K. Żochowski
- * New types of hydraulic pumps and motors prepared by the Chair of Hydraulics and Pneumatics, Gdańsk University of Technology, for practical applications by A. Balawender, L. Osiecki

from Technical University of Szczecin

- * Carbon nanoforms a new material in material science by R.J.Kaleńczuk, E. Borowiak-Palen
- Evaluation of geothermal heat utilization possibilities in heat plant cooperating with heat receivers connected in parallel – by W. Nowak, K. Zwarycz

from the State Higher Engineering School of Elblag

★ Dynamic analysis of high-power turboset – by M. Kahsin, H. Olszewski, Z.Walczyk.

ASME - TURBO EXPO 2004

From 14 to 17 June 2004 a global gas turbine event for education, technology and networking had place in Vienna. It was the international Technical Congress of American Society of Mechanical Engineering (ASME) under the heading:

Power for Land, Sea and Air

During 214 sessions were presented papers prepared by almost 2000 specialists representing universities, scientific institutes, scientific research centres and industrial enterprises of many countries of the world. In the event also Polish scientific workers took part. They presented 9 papers as follows:

- ❖ Mathematical model of Solid Oxide Fuel Cell (SOFC) for power plant simulations – by A. Miller, J. Milewski (Warsaw University of Technology)
- Optimization of rotor critical speeds by change of features of machine's bearings – by J. Rybczyński (Institute of Fluid-Flow Machinery, Gdańsk, Polish Academy of Sciences)
- Experimental analysis and prediction of wake-induced transition in turbomachinery – by W. Piotrowski, S. Drobniak, W. Elsner (Technical University of Częstochowa) and S. Vilmin (Numeca International, Belgium)

- ❖ Natural frequencies and modes shapes of two tuned and mistuned bladed discs on the shaft – by R. Rządkowski, M. Drewczyński (Institute of Fluid-Flow Machinery, Gdańsk, Polish Academy of Sciences)
- Unsteady load of partial admission control stage rotor of a large power steam turbine – by R. Rządkowski, P. Lampart, M. Szymaniak (Institute of Fluid-Flow Machinery, Gdańsk, Polish Academy of Sciences)
- Tip leakage/main flow interactions in multi-stage HP turbines with short-height blading – by M. Szymaniak, P. Lampart (Institute of Fluid-Flow Machinery, Gdańsk, Polish Academy of Sciences) and S. Yershov, A. Rusanov (Institute for Mechanical Engineering Problems, Ukraine)
- Application of ANN for diagnostics of the geometry deteriorations of the power system apparatuses by J. Głuch (Gdańsk University of Technology) and J. A. Krzyżanowski (Institute of Fluid-Flow Machinery, Gdańsk, Polish Academy of Sciences)
- An impact of environmental disturbances on combined cycle power plant control – by Z. Domachowski, M. Dzida (Gdańsk University of Technology)
- Expansion line modelling and strength diagnostics of internally cooled gas turbines – by W. Kosman, T. Chmielniak (Silesian University of Technology, Gliwice).